12. Equilibrium Measures on Recurrent Markov Processes

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(Communicated by Kôsaku YOSIDA, M. J. A., Feb. 12, 1985)

1. Introduction. We consider the potential theory for recurrent Markov processes introduced by T. Ueno [4]. He studied a pair of measures μ_L^K and μ_K^L satisfying $\mu_L^K(\cdot)=\mu_K^L h_K(\cdot)$, $\mu_K^L(\cdot)=\mu_L^K h_L(\cdot)$, where $h_K(x,\cdot)$ is the hitting measure to the set K. In this paper we prove that in the symmetric case the measure ν_L^K multiplied μ_L^K by the Ueno capacity is the equilibrium measure on $K\subset L^c$. Further we show that the equilibrium potential induced by ν_L^K is the hitting probability for K before attaining to L. We anticipate that such a pair of measures μ_L^K and μ_K^L is a new probabilistic characterization of the equilibrium measure.

2. Preliminaries. We refer the reader to [2] for all terminology and notation not explicitly defined here. Let R be a separable Hausdorff locally compact space containing at least two points and satisfying

(R.1) For each point $x\in R$, we can take a countable base of neighborhoods of x consisting of arcwise connected open sets,

(R.2) R is connected.

We denote by $\boldsymbol{B}$ the topological Borel field of subsets of R. For a set $A\in\boldsymbol{B}$ and a path function $X(t)$ from $[0,\infty)$ to R, σ_A is defined by

$$\begin{aligned}\sigma_A&=\inf\{t\geqq 0\,|\,X(t)\in A\}, && \text{if such } t \text{ exists},\\ &=\infty, && \text{otherwise.}\end{aligned}$$

We denote by $\mathscr{B}$, the smallest Borel field of subsets of the sample space W containing $\{w\,|\,X(t,w)\in A\}$ for all $A\in\boldsymbol{B}$ and $t\geqq 0$. Let $\{P_x(\cdot), x\in R\}$ be a system of probability measures on satisfying

(P.1) $P_x(E)$ is a $\boldsymbol{B}$-measurable function of x for each $E\in\mathscr{B}$,

(P.2) $P_x(\{w\,|\,X(0,w)=x\})=1$ for each $x\in R$,

(P.3) quasi-left continuity,

(P.4) Markov property.

In order to study a broad class of recurrent Markov process Ueno [4] introduced the following assumptions (X.1)$\sim$(X.5) which we follow.

(X.1) Recurrence: $P_x(X(t)\in A$ for some $0\leqq t<\infty)=1$ for any $x\in A$, $A\in\boldsymbol{B}$.

We define the hitting measure $h_A(x,\cdot)$ for the set $A\in\boldsymbol{B}$ by

$$h_A(x,E)=P_x(X(\sigma_A)\in E,\ \sigma_A<\infty),\quad x\in R,\quad E\in\boldsymbol{B}.$$

(X.2) For any continuous function f on A,

$$h_Af(x)=\int h_A(x,dy)f(y)$$

is continuous in A^c, where A is a closed set in R containing an inner point.