11. The Robin Problems on Domains with Many Tiny Holes

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(Communicated by Kôsaku YOSIDA, M. J. A., Feb. 12, 1985)

1. Introduction. Let Ω be a bounded domain of $\boldsymbol{R}^N$, $N \geqq 2$, with smooth boundary Γ and with the outer unit normal vector ν_Γ on Γ. Let $\boldsymbol{R}^N$ be divided into an infinitely many number of cubes C_i^ε, $i \in N$, with volume of $(2\varepsilon)^N$ and let $B_i(r^\varepsilon)$ be a closed ball of radius r^ε $(<\varepsilon)$ centered in C_i^ε. From Ω we remove all such balls and obtain a $D^\varepsilon(\subset \Omega)$ with n^ε holes. Under the case $n^\varepsilon \to \infty$, $r^\varepsilon \to 0$, a parameter, which determines the behavior of the Laplacian on D^ε with the Dirichlet condition, is known by M. Kac [2], J. Rauch and M. Taylor [3], D. Cioranescu and F. Murat [1]. That parameter is given by $\lim n^\varepsilon (r^\varepsilon)^{N-2}$ for $N \geqq 3$ and $\lim n^\varepsilon / |\log r^\varepsilon|$ for $N=2$, ε means values of a fixed sequence decreasing to zero. Now, we show a different parameter is important for the Robin problems.

From Ω we remove all balls $B_i(r^\varepsilon)$ such that dist $(B_i(r^\varepsilon), \Gamma) \geqq \varepsilon$ and obtain R^ε with n^ε holes. Let α be a positive constant and ν^ε the outer unit normal vector on ∂R^ε. We consider the Robin problem: for $f \in L^2(\Omega)$ find $u^\varepsilon \in H^1(R^\varepsilon)$ such that

$$(1) \qquad \begin{aligned} &-\Delta u^\varepsilon = f \qquad \text{a.e. in } R^\varepsilon, \\ &\frac{\partial u^\varepsilon}{\partial \nu^\varepsilon} + \alpha u^\varepsilon = 0 \qquad \text{a.e. on } \partial R^\varepsilon. \end{aligned}$$

Theorem 1. *Let u^ε be the solution of* (1) *and $\tilde{u}^\varepsilon \in H^1(\Omega)$ an extension of u^ε to be harmonic in F^ε, $F^\varepsilon = \Omega \backslash R^\varepsilon$. Assume that $r^\varepsilon \to 0$ and $n^\varepsilon \to \infty$ with the conditions $\eta = \lim n^\varepsilon (r^\varepsilon)^{N-1}$, $0 < \eta < \infty$. Then $\tilde{u}^\varepsilon$ converges weakly in $H^1(\Omega)$ to the solution of the problem:*

$$(2) \qquad \begin{aligned} &-\Delta u + \frac{\alpha S_N \eta u}{|\Omega|} = f \qquad \text{a.e. in } \Omega, \\ &\frac{\partial u}{\partial \nu_\Gamma} + \alpha u = 0 \qquad \text{a.e. on } \Gamma. \end{aligned}$$

Here $|\Omega|$ means the volume of Ω and S_N means the surface area of the unit sphere of $\boldsymbol{R}^N$.

2. Abstract scheme. Let Ω be the same domain as in Section 1. We introduce a certain limit of the minus Laplacian, which corresponds to one of versions for theorem 1.2 of Cioranescu-Murat [1] in the case of Robin condition.

For a subdomain G of Ω we regard all functions of $L^2(G)$ as functions of $L^2(\Omega)$ vanishing outside G. In this section R^ε means a subdomain of Ω satisfying (a.1) below. Let $a^\varepsilon : H^1(R^\varepsilon) \times H^1(R^\varepsilon) \to \boldsymbol{R}$ be a bilinear form defined by