96. *Infinitely Many Periodic Solutions for a Superlinear Forced Wave Equation*

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1. Introduction. In this article we shall study the nonlinear wave equation:

(1) $v_{tt}-v_{xx}+g(v)=f(x,t),\qquad (x,t)\in(0,\pi)\times \boldsymbol{R},$

(2) $v(0,t)=v(\pi,t)=0,\qquad t\in \boldsymbol{R},$

(3) $v(x,t+2\pi)=v(x,t),\qquad (x,t)\in(0,\pi)\times \boldsymbol{R},$

where $g\in C(\boldsymbol{R},\boldsymbol{R})$ is a function such that $g(\xi)/\xi\to\infty$ as $|\xi|\to\infty$ and $f(x,t)$ is a 2π-periodic function of t.

In a previous paper K. Tanaka [5] we studied (1)–(3) in case $g(\xi)=\pm|\xi|^{s-1}\xi$. This paper is a continuation of [5] and deals with more general equations. Our main result is as follows:

Theorem. *Suppose that* $g\in C(\boldsymbol{R},\boldsymbol{R})$ *satisfies*

(g_1) $g(\xi)$ *is strictly increasing,*

(g_2) *there exist* $\mu>2$ *and* $l\geqq 0$ *such that for* $|\xi|\geqq l$,

$$0<\mu G(\xi)\equiv\mu\int_0^\xi g(\tau)d\tau\leqq \xi g(\xi),$$

(g_3) *there exist* $s>1$ *and* $C>0$ *such that for* $\xi\in\boldsymbol{R}$,

$$|g(\xi)|\leqq C(|\xi|^s+1),$$

(g_4) $\dfrac{2}{s-1}>\dfrac{\mu}{\mu-1}$.

Then, for all 2π*-periodic* $f(x,t)\in L^\infty([0,\pi]\times\boldsymbol{R})$, *there exists an unbounded sequence of weak solutions of* (1)–(3) *in* L^∞.

In [3], P. H. Rabinowitz obtained the conditions which ensure the existence of an unbounded sequence of solutions of the semilinear elliptic equation:

$$-\Delta u=g(u)+f(x),\qquad x\in D,$$
$$u=0,\qquad x\in\partial D,$$

where $D\subset\boldsymbol{R}^n$ is a smooth bounded domain. In particular, in case $n=2$, his conditions are (g_2), (g_3), (g_4) and

(g_5) $g(-\xi)=-g(\xi)\qquad$ for all $\xi\in\boldsymbol{R}$.

He also obtained a similar existence result for the second order Hamiltonian systems of ordinary differential equations. For the wave equation (1)–(3), we act on S^1-symmetry and get the existence result without assumption (g_5).

As in K. Tanaka [5], we use a perturbation result of P. H. Rabinowitz [3] asserting the existence of infinitely many critical points of perturbed