93. *Stability Theorem for Singularly Perturbed Solutions to Systems of Reaction-Diffusion Equations*

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(Communicated by Kôsaku YOSIDA, M. J. A., Dec. 12, 1985)

§1. **Introduction.** This note presents a stability theorem to singularly perturbed stationary solutions (SPS) of the systems of nonlinear diffusion equations with a small parameter $\varepsilon>0$:

$$\text{(P)}\quad \begin{array}{l} u_t=\varepsilon^2 u_{xx}+f(u,v) \quad\text{and}\quad v_t=Dv_{xx}+g(u,v), \quad x\in I=(0,1),\ t>0,\\ u_x=0=v_x, \qquad x\in\partial I=\{0,1\}. \end{array}$$

The existence problem of SPS has a rather long history, see, for instance, [3]. For the stability properties of SPS, however, very few works have been known (see [2]). An exception is the work for degenerate case $\varepsilon=0$ of a simple density-dependent diffusion system ([1]). Recent works of the authors ([7] and [8]) show the stability of SPS for large D, where the basic method is a perturbation from the limit of $D\uparrow+\infty$. However, the stability of SPS for a general D has remained open up to the present time. In this note, we give a new idea to solve the stability problem of SPS of one mode type (SPS1) for a general D, where *the singular limit eigenvalue problem* plays a key role. Let us state the main assumptions for f and g. They are smooth functions defined on an open set $\mathcal{O}$ in $\boldsymbol{R}^2$ such that

(A.1) The nullcline of f is sigmoidal, and consists of three curves $u=h_-(v)$, $h_0(v)$, and $h_+(v)$ with $h_-(v)<h_0(v)<h_+(v)$.

(A.2) $J(v)$ has an isolated zero at $v=v^*$ such that $dJ/dv<0$ at $v=v^*$, where

$$J(v)=\int_{h_-(v)}^{h_+(v)} f(s,v)ds.$$

(A.3) Let

$$G(v)=\begin{cases} g(h_-(v),v), & v\leqq v^*\\ g(h_+(v),v), & v\geqq v^*. \end{cases}$$

Then $dG/dv<0$. Moreover, $g>0$ on the curve $\mathcal{C}_+: u=h_+(v)$ for $v\geqq v^*$, and $g<0$ on $\mathcal{C}_-: u=h_-(v)$ for $v\leqq v^*$. Also, $f_u<0$ on $\mathcal{C}_+\cup\mathcal{C}_-$.

(A.4) (Stability Assumption) On $\mathcal{C}_+\cup\mathcal{C}_-$, $g_v<0$.

For the definitions of function spaces $H^k(I)$, $H^k_N(I)$, and $C^2_\varepsilon(I)$, see [5] and [6]. Under (A.1)–(A.3), the following result is known.

Existence Theorem of SPS1 (Mimura-Tabata-Hosono [5] and Ito [4]). *Suppose there exists a monotone increasing solution $V=V^*(x)$ of $DV_{xx}+G(V)=0$ in I with $V_x=0$ on ∂I, for a given $D>0$. Then, there exists a constant $\varepsilon_0>0$ such that* (P) *has an ε-family of* SPS1 $U^\varepsilon=(u(x;\varepsilon), v(x;\varepsilon))$ *for $0<\varepsilon<\varepsilon_0$. U^ε is uniformly bounded in $C^2_\varepsilon\times C^2$, and satisfies*

$$\lim_{\varepsilon\downarrow 0} u(x;\varepsilon)=U^*(x)\stackrel{\text{def}}{=}\begin{cases} h_-(V^*(x)), & x\in[0,x^*)\\ h_+(V^*(x)), & x\in(x^*,1], \end{cases}$$