91. Jacobian Rings of Hypersurfaces of Compact Irreducible Hermitian Symmetric Spaces and Generic Torelli Theorem

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0. The purpose of the present notes is to show some properties of Jacobian rings of smooth hypersurfaces of compact irreducible Hermitian symmetric spaces, in particular, the duality properties and the symmetrizer lemma. Using these properties, we shall prove the generic Torelli theorem for smooth hypersurfaces with the ample canonical bundles in such Hermitian symmetric spaces with a few exceptions (Theorem 4.1). In case of the projective space $\boldsymbol{P}^n$, the Jacobian ring of a smooth hypersurface is an *Artinian Gorenstein ring* by the local duality theorem (cf. [5]). Using this property, Donagi proved the *symmetrizer lemma* for projective hypersurfaces, which is a key step in his proof of the generic Torelli theorem for projective hypersurfaces (cf. [2], see also [8] for the weighted projective case). Contrary to the projective case, the Jacobian rings here are *not* Gorenstein in general and we shall give a sufficient condition for the duality property (DP) (Theorem 1.2, Theorem 2.1). By using this, we shall show the symmetrizer lemma, which is sufficient to prove our generic Torelli theorem. Details will be published elsewhere.

1. Let G be a complex simple Lie group and let U be a parabolic Lie subgroup such that the quotient manifold $Y=G/U$ is a compact irreducible Hermitian symmetric space. These Hermitian symmetric spaces are divided into six classes; I. $Gr(m+n, n)=U(m+n)/U(m)\times U(n)$, II. $SO(2n)/U(n)$, III. $Sp(n)/U(n)$, IV. $Q^n=SO(n+2)/SO(2)\times SO(n)$ $(n>2)$, V. $E_6/Spin(10)\times T^1$, VI. $E_7/E_6\times T^1$, (see, e.g., [1]). Let $H=\mathcal{O}_Y(1)$ denote the ample generator of the Picard group $\mathrm{Pic}(Y)\cong\boldsymbol{Z}$ and we shall write $\mathcal{O}_Y(a)$ instead of $H^{\otimes a}$.

Let $X\subset Y$ be a *smooth hypersurface of degree* d, defined by a section $f\in H^0(\mathcal{O}_Y(d))$ and let $S=\underset{a\geqq 0}{H^0}(\mathcal{O}_Y(a))=\bigoplus_{a\geqq 0} S^a$ denote the homogeneous coordinate ring of Y and $\mathfrak{g}$ the Lie algebra of G.

Definition 1.0. Let Y be a compact Hermitian symmetric space which is not isomorphic to $\boldsymbol{P}^n$. The *Jacobian ideal* J_f of the smooth hypersurface X is a homogeneous ideal of S generated by $\mathfrak{g}\cdot f=\{v\cdot f\in S^d\mid v\in\mathfrak{g}\}$ and $f\in S^d$. The *Jacobian ring* $R=R_f$ is the quotient $R_f=S/J_f$.

Let Y, X, S, $f\in S^d$ and $R=R_f$ be as above. A positive integer $\lambda=\lambda(Y)$

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