8. *Propagation of Wave Front Sets of Solutions of the Cauchy Problem for a Hyperbolic System in Gevrey Classes*

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Introduction and main theorem. Consider a hyperbolic system

$$\mathcal{L}=D_t-\begin{pmatrix}\lambda_1(t,X,D_x) & & 0\\ & \ddots & \\ 0 & & \lambda_l(t,X,D_x)\end{pmatrix}+(b_{jk}(t,X,D_x)) \quad \text{on } [0,T]\times R^n_x \tag{1}$$

with real symbols $\lambda_j(t,x,\xi)$ in $G^{(\kappa)}([0,T];S^1_{G(\kappa)})$ and symbols $b_{jk}(t,x,\xi)$ in $G^{(\kappa)}([0,T];S^\sigma_{G(\kappa)})$ $(0\leqq\sigma<1/\kappa)$. Here, for $\kappa>1$ we denote by $G^{(\kappa)}([0,T];S^m_{G(\kappa)})$ a class of symbols $p(t,x,\xi)$ of pseudo-differential operators satisfying

$$|\partial_t^\gamma\partial_\xi^\alpha\partial_x^\beta p(t,x,\xi)|\leqq CM^{-(\gamma+|\alpha|+|\beta|)}\gamma!^\kappa\alpha!^\kappa\beta!^\kappa\langle\xi\rangle^{m-|\alpha|}$$

for constants C and M. In the recent paper [9] the second author has constructed the fundamental solution of (1) assuming the constant multiplicities of characteristic roots of $\mathcal{L}$ and investigated the propagation of wave front sets for the solution of the Cauchy problem of $\mathcal{L}$:

$$\mathcal{L}U(t)=0 \quad (0<t\leqq T_0), \quad U(0)=G. \tag{2}$$

In the present paper we study the propagation of wave front sets in Gevrey classes for the solution $U(t)$ of (2) without assuming the constant multiplicity and get a similar result to the one for the C^∞ case obtained by Kumano-go and the second author [4].

Let $\varepsilon>0$ and let V be a conic set in $T^*(R^n_x)$. Then, we denote by $\Gamma^\nu_\varepsilon(t,V)$ $(\nu=0,1,\cdots)$ the set of end points (at t) of all ε-admissible trajectories of, at most, step ν issuing from the ε-conic neighborhood $V_\varepsilon\equiv\{(x,\xi);|x-y|\leqq\varepsilon, |\xi/|\xi|-\eta/|\eta||\leqq\varepsilon, (y,\eta)\in V\}$ of V (concerning the characteristic roots $\lambda_j(t,x,\xi)$, $j=1,\cdots,l$; cf. [2]) and set

$$\begin{cases}\Gamma_\varepsilon(t,V)=\textit{the closure of } \bigcup_{\nu=0}^\infty\Gamma^\nu_\varepsilon(t,V),\\ \Gamma(t,V)=\bigcap_{\varepsilon>0}\Gamma_\varepsilon(t,V).\end{cases} \tag{3}$$

We also denote by $\mathcal{D}^{\{\kappa\}\prime}_{L^2}$ a class of ultradistributions defined in [3] (see also [11]).

Theorem. *Let $\mathcal{L}$ be a hyperbolic operator of the form* (1) *with $\lambda_j(t,x,\xi)\in G^{(\kappa)}([0,T];S^1_{G(\kappa)})$ and $b_{jk}(t,x,\xi)\in G^{(\kappa)}([0,T];S^\sigma_{G(\kappa)})$ for $0\leqq\sigma<1/\kappa$. Consider the Cauchy problem* (2). *Then, there exists a unique solution $U(t)$ in $\mathcal{B}^1([0,T_0];\mathcal{D}^{\{\kappa\}\prime}_{L^2})$ $(0<T_0\leqq T)$ for any $G\in\mathcal{D}^{\{\kappa\}\prime}_{L^2}$ and it satisfies*

$$\mathrm{WF}_{G(\kappa_1)}(U(t))\subset\Gamma(t,\mathrm{WF}_{G(\kappa_1)}(G)) \tag{4}$$

for any κ_1 satisfying $\kappa\leqq\kappa_1<1/\sigma$.

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