75. Dynkin Graphs and Combinations of Singularities on Quartic Surfaces

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In this article we show that possible combinations of singularities on quartic surfaces in the three-dimensional projective space $\boldsymbol{P}^3$ can be described systematically by Dynkin graphs. Details of the proof will appear elsewhere. We assume that every variety is algebraic and is defined over the complex number field $\boldsymbol{C}$.

Definition 1. A disjoint finite union of connected Dynkin graphs of type A, B, D or E is called a Dynkin graph. For a Dynkin graph, the following procedure is called an *elementary transformation* of it.

(1) Replace each component by the extended Dynkin graph of the corresponding type.

(2) Choose in an arbitrary manner at least one vertex from each component (of the extended Dynkin graph) and then remove these vertices together with the edges issuing from them (cf. Bourbaki [1]).

Note that any connected Dynkin graph of type A, D, or E corresponds to a singularity on a surface (cf. Durfee [2]).

Theorem 2. *Let* $G=\sum_{k\geq 1} a_k A_k+\sum_{l\geq 4} b_l D_l+\sum_{m=6}^{8} c_m E_m$ *(a finite sum) be a Dynkin graph with only components of type A, D or E. Set* $r=\sum a_k k+\sum b_l l+\sum c_m m$. *Then the following conditions (A) and (B) are equivalent.*

(A) There exists a quartic surface in the projective space of dimension 3 whose combination of singularities just agrees with G and moreover one of the following conditions $\langle 1\rangle$, $\langle 2\rangle$, $\langle 3\rangle$, $\langle 4\rangle$ *holds for the root lattice* $Q=Q(G)$ *of type G.*

$\langle 1\rangle$ $r=17$, *the discriminant* $d(Q)$ *of* Q *is a square number, and for every prime number* p, $\varepsilon_p(Q)=1$.

$\langle 2\rangle$ $r=16$, *and for every prime number* p, $\varepsilon_p(Q)=(-1, d(Q))_p$.

$\langle 3\rangle$ $r=15$, *and for every prime number* p, $-d(Q)\notin \boldsymbol{Q}_p^{*2}$ *or* $\varepsilon_p(Q)=(-1,-1)_p$.

$\langle 4\rangle$ $r\leq 14$.

(B) G coincides with a Dynkin graph which is obtained from one of the following 9 basic Dynkin graphs by elementary transformations repeated twice such that it has no vertices corresponding to short roots.

$$B_{17},\ D_{16},\ D_{12}+B_5,\ A_{15}+B_2,\ A_{11}+E_6,\ 2D_8,\ 2E_8,\ E_8+B_9,\ 2E_7+B_3.$$

Remarks. 1. $r=\operatorname{rank} Q=$ the number of vertices in G.

2. The symbol $\varepsilon_p(Q)\in\{+1,-1\}$ denotes the Hasse symbol of the inner product space $Q\otimes\boldsymbol{Q}$ over $\boldsymbol{Q}$. The symbol $(\ ,\ )_p$ is the Hilbert symbol. $\boldsymbol{Q}_p$