73. On Sufficient Conditions for Convergence of Formal Solutions

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§ 1. Introduction. Let $x=(x_1, x_2) \in \boldsymbol{C}^2$. For a multi-index $\alpha=(\alpha_1, \alpha_2) \in \boldsymbol{N}^2$, $\boldsymbol{N}=\{0, 1, 2, \cdots\}$, we set $(x\cdot\partial)^\alpha=(x_1\cdot\partial_1)^{\alpha_1}(x_2\cdot\partial_2)^{\alpha_2}$ where $\partial=(\partial_1\cdot\partial_2)$, $\partial_j=\partial/\partial x_j$, $j=1, 2$. Let $m\geqq 0$, $N\geqq 1$, $s\geqq 0$ be integers such that $0\leqq s\leqq m$ and let $s_1, \cdots, s_N$ be a set of integers such that $1=s_1\leqq s_2\leqq \cdots \leqq s_N$. In this note we are concerned with the convergence of all formal solutions of the equation

$$(P_0(x\cdot\partial)+Q_s(x\,;x\cdot\partial))u=f \tag{1.1}$$

where u denotes ${}^t(u_1, \cdots, u_N)$, $f={}^t(f_1, \cdots, f_N)$ is a given analytic vector function and the operators P_0 and Q_s are given by

$$P_0(x\cdot\partial)=\Big(\sum_{|\alpha|=m+s_j-s_k} a_\alpha^{jk}(x\cdot\partial)^\alpha\Big)_{\substack{j\downarrow 1,\dots,N\\ k\to 1,\dots,N}} \tag{1.2}$$

$$Q_s(x\,;x\cdot\partial)=\Big(\sum_{|\beta|\leqq m-s+s_j-s_k} b_\beta^{jk}(x)(x\cdot\partial)^\beta\Big)_{\substack{j\downarrow 1,\dots,N\\ k\to 1,\dots,N}}. \tag{1.3}$$

Here $a_\alpha^{jk}\in \boldsymbol{C}$ and $b_\beta^{jk}(x)$ are analytic at $x=0$. If $s=0$, then we may assume that $b_\beta^{jk}(0)=0$ ($|\beta|=m+s_j-s_k$) in (1.1). Hence we assume this from now on.

Concerning this problem Kashiwara-Kawai-Sjöstrand showed the convergence of all formal solutions for a wider class of equations than (1.1) under the so-called ellipticity condition (cf. [2]). Here we show a new phenomenon when the ellipticity condition is not satisfied for equations belonging to a subclass of equations studied in [2]. Namely we shall introduce a new diophantine function $F_\sigma(t)$ and give a sufficient condition for the convergence of all formal solutions in terms of $F_\sigma(t)$. We note that this result is applied to the problem of holomorphic prolongation of solutions across characteristic points.

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§ 2. Notations and results. For $R>0$, $d\geqq 0$ let us define the set $\Gamma_{R,d}$ of holomorphic functions by

$$\Gamma_{R,d}=\{h(x)=\sum_{\gamma\geqq 0} h_\gamma x^\gamma/\gamma\,!\,;\ K>0 \text{ independent of } \gamma \text{ such that } |h_\gamma|\leqq K|\gamma|!\ R^{-|\gamma|}(|\gamma|+1)^{-d}\} \tag{2.1}$$

where $|h_\gamma|$ denotes the usual maximal norm of N-dimensional vector h_γ. For $\sigma\geqq 0$ we define the function $F_\sigma(t)$ of $t\in \boldsymbol{C}$ by

$$F_\sigma(t)=\{\text{the set of all the cluster values of the sequence } \{\mu^\sigma(\nu/\mu-\tau)\} \text{ when } \nu, \mu\in \boldsymbol{N} \text{ and } \nu, \mu\to\infty\}. \tag{2.2}$$

Remark. Obviously the function $F_\sigma(t)$ is multivalued in general. Here we list up some of its fundamental properties without proofs. The