72. Asymptotic Expansions of Solutions of Fuchsian Hyperbolic Equations in Spaces of Functions of Gevrey Classes

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In this paper, we deal with Fuchsian hyperbolic equations with Gevrey coefficients and establish the asymptotic expansions of solutions in spaces of functions of Gevrey classes. As to the Cauchy problem in Gevrey classes, see Tahara [3].

1. Fuchsian hyperbolic equations. Let us consider the following equation:

(E) $$(t\partial_t)^m u+\sum_{\substack{j+|\alpha|\leqq m\\ j<m}} t^{l(j,\alpha)}a_{j,\alpha}(t,x)(t\partial_t)^j\partial_x^\alpha u=0,$$

where $(t, x)=(t, x_1, \cdots, x_n)\in[0, T]\times \boldsymbol{R}^n$ $(T>0)$, $m\in \boldsymbol{N}(=\{1, 2, \cdots\})$, $\alpha=(\alpha_1, \cdots, \alpha_n)\in \boldsymbol{Z}_+^n(=\{0, 1, 2, \cdots\}^n)$, $|\alpha|=\alpha_1+\cdots+\alpha_n$, $l(j, \alpha)\in \boldsymbol{Z}_+$ $(j+|\alpha|\leqq m$ and $j<m)$, $a_{j,\alpha}(t, x)\in C^\infty([0, T]\times \boldsymbol{R}^n)$ $(j+|\alpha|\leqq m$ and $j<m)$, $\partial_t=\partial/\partial t$, and $\partial_x^\alpha=(\partial/\partial x_1)^{\alpha_1}\cdots(\partial/\partial x_n)^{\alpha_n}$. Assume the following conditions:

(A-1) $l(j, \alpha)\in \boldsymbol{Z}_+$ $(j+|\alpha|\leqq m$ and $j<m)$ satisfy

$$\begin{cases} l(j,\alpha)=\kappa_1\alpha_1+\cdots+\kappa_n\alpha_n, & \text{when } j+|\alpha|=m \text{ and } j<m,\\ l(j,\alpha)>0, & \text{when } j+|\alpha|<m \text{ and } |\alpha|>0,\\ l(j,\alpha)\geqq 0, & \text{when } j+|\alpha|<m \text{ and } |\alpha|=0 \end{cases}$$

for some $\kappa_1, \cdots, \kappa_n\in \boldsymbol{Q}$ such that $\kappa_i>0$ $(i=1, \cdots, n)$.

(A-2) All the roots $\lambda_i(t, x, \xi)$ $(i=1, \cdots, m)$ of

$$\lambda^m+\sum_{\substack{j+|\alpha|=m\\ j<m}} a_{j,\alpha}(t,x)\lambda^j\xi^\alpha=0$$

are *real*, *simple* and *bounded* on $\{(t, x, \xi)\in[0, T]\times \boldsymbol{R}^n\times \boldsymbol{R}^n; |\xi|=1\}$.

Then, (E) is one of the most fundamental examples of Fuchsian hyperbolic equations. The characteristic exponents $\rho=\rho_1(x), \cdots, \rho_m(x)$ are defined by the roots of

$$\rho^m+a_{m-1}(x)\rho^{m-1}+\cdots+a_0(x)=0,$$

where $a_j(x)=[t^{l(j,(0,\cdots,0))}a_{j,(0,\cdots,0)}(t,x)]|_{t=0}$ $(j=0, \cdots, m-1)$.

2. Asymptotic expansions in $C^\infty((0, T), \mathcal{E}(\boldsymbol{R}^n))$. Let $\mathcal{E}(\boldsymbol{R}^n)$ be the Schwartz space on $\boldsymbol{R}^n$ and let $C^\infty((0, T), \mathcal{E}(\boldsymbol{R}^n))$ be the space of all C^∞ functions on $(0, T)$ with values in $\mathcal{E}(\boldsymbol{R}^n)$. Then, by applying the result in Tahara [2] we have

Theorem 1. *Assume that* (A-1), (A-2) *and the condition*:

(T) $\quad l(j, \alpha)\geqq\kappa_1\alpha_1+\cdots+\kappa_n\alpha_n,$ *when* $j+|\alpha|<m$ *and* $|\alpha|>0$

hold, and that $\rho_i(x)-\rho_j(x)\notin \boldsymbol{Z}$ *holds for any* $x\in \boldsymbol{R}^n$ *and* $1\leqq i\neq j\leqq m$. *Then, we have the following results.*

(1) *Any solution* $u(t, x)\in C^\infty((0, T), \mathcal{E}(\boldsymbol{R}^n))$ *of* (E) *can be expanded*