71. The Regularity of Discrete Models of the Boltzmann Equation

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The aim of this note is to single out a category of the discrete Boltzmann equations as the regular models of the Boltzmann equation. It is shown that there exist regular models with n moduli of velocities for an arbitrary integer $n\geq 2$.

1. Let $M=\{v_1, \cdots, v_d\}$ be the set of velocities, i.e., the constant vectors in $\boldsymbol{R}^3$. We assume that the linear span of M coincides with $\boldsymbol{R}^3$. The model M is essentially three-dimensional in this sense. First of all, we introduce the notion of the collision. Let us denote by Σ the set of all unordered pairs of distinct velocities. We may set

$$\Sigma=\{(v_i, v_j)\,;\, 1\leq i<j\leq d\}.$$

Let $\alpha, \beta\in\Sigma$. Then the ordered pair of α and β is called a collision, if

(i) $\alpha\neq\beta$,

(ii) the momentum of α equals the momentum of β,

(iii) the energy of α equals the energy of β.

It is usual to denote the ordered pair by $\alpha\rightarrow\beta$. We call α and β the initial and the final states of the collision, respectively. It is assumed in the following that there exists at least one collision. Now let C be the set of all collisions. We obtain a partition of C by the equivalence relation given below.

We introduce the group of transformations acting in M. We set

$$\tilde{G}=\{T\,;\, T\in\mathcal{O}(\boldsymbol{R}^3),\ TM=M\}.$$

Here, $\mathcal{O}(\boldsymbol{R}^3)$ denotes the group of orthogonal transformations. $\tilde{G}$ induces naturally a group of isometric transformations on M, which we denote by G. It is easily seen that G is determined uniquely as the maximal set of isometric transformations on M. We define that $\alpha\rightarrow\beta$ and $\alpha'\rightarrow\beta'$ are equivalent if these collisions are obtained from each other by performing a transformation which belongs to G or by interchanging the initial and the final states or by combining these two operations. The constants A_{ij}^{kl} appearing in the definition of the collision term may be identified with a "step function" subordinate to the partition of C, which is induced from the equivalence relation given above. (See [3] for details.) Thus, if C consists of m equivalence classes, we have m arbitrary constants in defining the collision term. The general form of the discrete Boltzmann equation is given by