65. A Construction of Lie Algebras and Lie Superalgebras by Freudenthal-Kantor Triple Systems. I

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1. Introduction. In our previous paper [3], we showed that from a two-dimensional associative triple system W and any generalized Jordan triple system $U(-1, 1)$ of second order (due to I. L. Kantor [4]) we can make a generalized Jordan triple system $W\otimes U(-1, 1)$ of second order which induces the Lie triple system, and that we have a Lie algebra as a standard embedding of the Lie triple system. In this paper, it is shown that Lie algebras and Lie superalgebras can be also constructed by Freudenthal-Kantor triple system $U(\varepsilon, \delta)$ ($\varepsilon=\pm 1$, $\delta=\pm 1$) which becomes a generalized Jordan triple system of second order in case $\varepsilon=-1$, $\delta=1$. We can make, namely, from the same associative triple system W as in [3] and any Freudenthal-Kantor triple system $U(\varepsilon, \delta)$ a Freudenthal-Kantor triple system $W\otimes U(\varepsilon, \delta)$ to which we can associate a Lie algebra and a Lie superalgebra as a standard embedding of a Lie triple system $W\otimes U(\varepsilon, \delta)\oplus \overline{W\otimes U(\varepsilon, \delta)}$, where $\overline{W\otimes U(\varepsilon, \delta)}$ is an isomorphic copy of $W\otimes U(\varepsilon, \delta)$. We assume that any vector space considered in this paper is finite-dimensional and the characteristic of the base field Φ is different from 2 or 3. The author wishes to express his hearty thanks to Prof. K. Yamaguti for his kind advice and encouragement.

2. A triple system A with a trilinear product $\{abc\}$ is called an *associative triple system* (ATS) if $\{ab\{cde\}\}=\{a\{bcd\}e\}=\{\{abc\}de\}=\{a\{dcb\}e\}$ for all elements $a, b, c, d, e \in A$ [6].

Let W be a two-dimensional triple system which has a basis $\{e_1, e_2\}$ such that

$$(1)\qquad \begin{aligned} &\{e_1e_1e_1\}=\alpha e_1, \quad \{e_1e_1e_2\}=\{e_1e_2e_1\}=\{e_2e_1e_1\}=\alpha e_2,\\ &\{e_1e_2e_2\}=\{e_2e_1e_2\}=\{e_2e_2e_1\}=\beta e_1, \quad \{e_2e_2e_2\}=\beta e_2, \end{aligned}$$

where $\alpha, \beta \in \Phi$. Then W is a commutative ATS and is also a Jordan triple system.

In the ATS W, we have

$$(2)\qquad l(a, b)l(c, d)=l(c, d)l(a, b),$$

$$(3)\qquad l(a, b)l(c, d)=l(l(a, b)c, d)=l(c, l(b, a)d),$$

where $l(a, b)c=\{abc\}$, for $a, b, c, d \in W$.

A *Freudenthal-Kantor triple system* (FSK) $U(\varepsilon, \delta)$ is a vector space with a triple product $\{xyz\}$ satisfying

$$(4)\qquad [L(x, y), L(u, v)]=L(L(x, y)u, v)+\varepsilon L(u, L(y, x)v),$$