63. *Construction of Certain Vector Valued Siegel Modular Forms of Degree two*

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(Communicated by Kunihiko KODAIRA, M. J. A., Sept. 12, 1985)

§ 1. **Introduction.** Let St be the standard representation of $GL(2, \boldsymbol{C})$ and $V(k, r)$ a representation space of $\det^k \otimes \mathrm{Sym}^r \mathrm{St}$. We denote the full Siegel modular group of degree two by Γ_2. A C^∞-Siegel modular form f of type (k, r) and of degree two is a $V(k, r)$ valued C^∞-function on the Siegel upper half plane H_2 of degree two satisfying the equation

$$f((AZ+B)(CZ+D)^{-1}) = (\det^k \otimes \mathrm{Sym}^r \mathrm{St})(CZ+D) f(Z)$$

$$\text{for all } M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \in \Gamma_2$$

and the usual growth rate condition (see Borel [2, § 7]). We denote by $M^\infty_{k,r}(\Gamma_2)$ the $\boldsymbol{C}$-vector space of all such functions. We put

$$M_{k,r}(\Gamma_2) = \{ f \in M^\infty_{k,r}(\Gamma_2) \mid f \text{ is holomorphic on } H_2 \}.$$

We shall explicitly construct $M_{k,2}(\Gamma_2)$ for even k and prove some congruences of eigenvalues of Hecke operators. Details of this paper are included in [8]. The author would like to thank Prof. R. Tsushima for communicating his paper [13] before publication and Prof. N. Kurokawa for his encouragement.

§ 2. **Construction of modular forms of type (*k*, 2).** Let S_2 be the $\boldsymbol{C}$-vector space of complex symmetric matrices of size two. The representation of $GL(2, \boldsymbol{C})$ defined via $A \to \det(G)^k G A {}^t G$ for $G \in GL(2, \boldsymbol{C})$ and $A \in S_2$ is equivalent to $\det^k \otimes \mathrm{Sym}^2 \mathrm{St}$. Henceforth, we put $V(k, 2) = S_2$. We denote by $M^\infty_k(\Gamma_n)$ the $\boldsymbol{C}$-vector space of C^∞-Siegel modular forms of degree n and weight k. Let $M_k(\Gamma_n)$ and $S_k(\Gamma_n)$ be subspaces of $M^\infty_k(\Gamma_n)$ consisting of holomorphic Siegel modular forms and of holomorphic cusp forms, respectively. We agree that $M_k(\Gamma_2) = \{0\}$ for a negative k. For a variable $Z = \begin{pmatrix} z_1 & z_3 \\ z_3 & z_2 \end{pmatrix}$ on H_2 we put

$$Y = \frac{1}{2i}(Z - \bar{Z}) \quad \text{and} \quad \frac{d}{dZ} = \begin{bmatrix} \dfrac{\partial}{\partial z_1} & \dfrac{1}{2} \cdot \dfrac{\partial}{\partial z_3} \\ \dfrac{1}{2} \cdot \dfrac{\partial}{\partial z_3} & \dfrac{\partial}{\partial z_2} \end{bmatrix}.$$

We define a differential operator $\nabla = \nabla_k$ acting on $M_k(\Gamma_2)$ by

$$\nabla f = \frac{k}{2\pi i}(2iY)^{-1} f + \frac{1}{2\pi i} \frac{d}{dZ} f.$$

By Shimura [11, (4.5)], we have $\nabla f \in M^\infty_{k,2}(\Gamma_2)$ for $f \in M_k(\Gamma_2)$. For $f \in M_k(\Gamma_2)$ and $g \in M_j(\Gamma_2)$, we put