# 57. On a Theorem of R. H. Martin on Certain Cauchy Problems for Ordinary Differential Equations*)

By Giovanni EMMANUELE
Department of Mathematics, University of Catania, Catania, Italy

(Communicated by Kôsaku YOSIDA, M. J. A., Sept. 12, 1985)

**1. Introduction.** Let $E$ be a real Banach space with norm $\|\cdot\|$ and $X$ be a locally closed and convex subset of $E$. If $B, C: [0, 1]\times X\to E$ are two (suitable) functions, we consider the following Cauchy problem

(CP) $\qquad \dot{x}=B(t, x)+C(t, x), \quad x(0)=x_0$

where $x_0\in X$.

In the paper [2] R. H. Martin obtained the existence of a local solution of (CP) under the following assumptions;

($C_1$) $B$ and $C$ are continuous and bounded in $[0, 1]\times X$;

($C_2$) $\langle x-y, B(t, x)-B(t, y)\rangle\le\omega(t, \|x-y\|)\|x-y\|$ for all $(t, x)$, $(t, y)$ in $[0, 1]\times X$, where $\omega(t, u)$: $[0, 1]\times[0, \infty)\to[0, \infty)$ is a continuous function such that $\omega(t, 0)=0$ for all $t\in[0, 1]$ and for which the Cauchy problem $\dot{u}=\omega(t, u)$, $u(0)=0$ has the unique solution $u(t)=0$ for all $t\in[0, 1]$;

($C_3$) $K$ is a relatively compact subset of $E$ such that $C(t, x)\in K$ for all $(t, x)\in[0, 1]\times X$;

($C_4$) $\liminf_{h\to+0} d(x+h(B(t, x)+C(t, x)); X)/h=0$ for all $(t, x)\in[0, 1]\times X$;

($C_5$) $C$ is uniformly continuous on $[0, 1]\times X$.

A diligent examination of the proof of this result shows the important role of the assumptions ($C_3$) and ($C_5$).

The hypothesis ($C_3$) plays a fundamental role also in other results contained in the same paper of Martin; however, recently (see [1]) it has been weakened using the following one;

($C_3$)′ there is a Lebesgue measurable subset $J$ of $[0, 1]$ with Lebesgue measure $m(J)=0$ for which $C(t, X)$ is relatively compact for any $t\in J^c$ ($J^c$ denotes the complement of $J$ in $[0, 1]$)

in the setting of Gelfand-Phillips spaces, so improving certain results of [2].

Purpose of this note is to generalize the above cited result of Martin in general Banach spaces using ($C_3$)′ instead of ($C_3$).

**2. The existence results.** This section contains the announced generalization of Martin's theorem. Together ($C_3$)′ we shall also use the following other assumptions;

($C_1$)′ $B+C$ is continuous on $[0, 1]\times X$ and $B$ and $C$ are both bounded

*) Work performed under the auspices of G.N.A.F.A. of C.N.R. and partially supported by M.P.I. of Italy (40%, 1983).