49. Universal Central Extensions of Chevalley Algebras over Laurent Polynomial Rings and GIM Lie Algebras

By Jun MORITA*) and Yōji YOSHII**)

(Communicated by Kôsaku YOSIDA, M. J. A., June 11, 1985)

We will give an explicit description of the universal central extensions of Chevalley algebras over Laurent polynomial rings with n variables, which is a natural generalization of the result for $n=1$ established in Garland [1], and which is obtained in a different way from Kassel [4]. Using this, we will discuss about a certain class of GIM Lie algebras which are introduced by Slodowy [5] as a generalization of Kac-Moody Lie algebras.

1. Central extensions of Chevalley algebras. Let F be a field of characteristic zero. For a finite dimensional split simple Lie algebra $\mathfrak{g}$ over F and an F-algebra R, we will write $\mathfrak{g}(R)=R\otimes_F \mathfrak{g}$ and view $\mathfrak{g}(R)$ as a Lie algebra over F. Since $\mathfrak{g}(R)=[\mathfrak{g}(R), \mathfrak{g}(R)]$, there is a unique, up to isomorphism, universal central extension of $\mathfrak{g}(R)$. A central extension of $\mathfrak{g}(R)$:

(1) $$0\longrightarrow V\longrightarrow \mathfrak{a}\longrightarrow \mathfrak{g}(R)\longrightarrow 0$$

can be reduced to a skew-symmetric F-bilinear mapping:

(2) $$\{\cdot,\cdot\}: R\times R\longrightarrow V$$

satisfying $\{u, vw\}+\{v, wu\}+\{w, uv\}=0$ for all $u, v, w\in R$ (cf. [1], [3]).

2. Laurent polynomial rings. We denote by $F[X_1^{\pm 1},\cdots,X_n^{\pm 1}]$ the ring of Laurent polynomials in $X_1,\cdots,X_n$ with coefficients in F. Let $\mathfrak{c}$ be the F-vector space with a basis $\{z_v^{(1)},\cdots,z_v^{(n-1)},z_0^{(n)}\mid v\in \boldsymbol{Z}^n\}$. We define an F-bilinear mapping $\{\cdot,\cdot\}_1$:

$$F[X_1^{\pm 1},\cdots,X_n^{\pm 1}]\times F[X_1^{\pm 1},\cdots,X_n^{\pm 1}]\longrightarrow \mathfrak{c}$$

by, for all $r_i, s_i\in \boldsymbol{Z}$ $(i=1,\cdots,n)$,

(3) $$\{X_1^{r_1}\cdots X_n^{r_n}, X_1^{s_1}\cdots X_n^{s_n}\}_1 = \begin{cases} \sum_{i=1}^{k-1}\dfrac{r_is_k-s_ir_k}{l_k}z_v^{(i)}+\sum_{j=k+1}^{n} r_jz_v^{(j-1)} & \text{if } l_k\neq 0,\ l_{k+1}=\cdots=l_n=0 \text{ for some } k,\\ \sum_{i=1}^{n} r_iz_0^{(i)} & \text{if } l_i=0 \text{ for all } i,\end{cases}$$

where $v=(l_1,\cdots,l_n)$ and $l_i=r_i+s_i$.

Theorem 1. *Let $\mathfrak{g}$ be a finite dimensional split simple Lie algebra over F. Then the mapping* (3) *determines a universal central extension of* $\mathfrak{g}(F[X_1^{\pm 1},\cdots,X_n^{\pm 1}])$.

Notice that the dimension of $\mathfrak{c}$ is one $(n=1)$; infinite $(n\geq 2)$.

3. n-fold extended Cartan matrices. Let $\mathfrak{h}$ be a Cartan subalgebra of $\mathfrak{g}$, and $\mathfrak{g}=\mathfrak{h}+\sum_{\alpha\in\Delta}\mathfrak{g}^\alpha$ the root space decomposition of $\mathfrak{g}$ with respect to

*) Institute of Mathematics, University of Tsukuba.
**) Komaba Junior High School attached to University of Tsukuba.