46. Virtual Characters and Constant Coefficient Invariant Eigendistributions on a Semisimple Lie Group

By Kyo NISHIYAMA

Department of Mathematics, Kyoto University

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Introduction. Let G be a connected semisimple Lie group with finite centre. For simplicity, we assume that G is acceptable. Take an irreducible quasi-simple representation (π, E) of G on a Hilbert space E. Then we can define a distribution $\Theta(\pi)$ on G, called an *irreducible character* of (π, E), as follows:

$$\Theta(\pi)(f) = \mathrm{Trace} \int_G f(g)\pi(g)dg,$$

where f is a C^∞-function on G with compact support, and dg is a Haar measure on G. The notion of irreducible characters plays an important role in representation theory of such groups. For example, they decide irreducible representations of G up to infinitesimal equivalence.

Harish-Chandra studied the properties of irreducible characters during 1950's and 1960's. His study was very elaborate one and he got many deep results. However, most of the results follow from the fact that characters are invariant eigendistributions on G.

An *invariant eigendistribution* (IED) is a distribution on G which has the following two properties.

(1) It is invariant under inner automorphisms of G.

(2) It is a simultaneous eigendistribution of two-sided invariant differential operators on G.

As commented above, irreducible characters are IEDs. Then, it is natural to ask if characters and IEDs are different notions. As is well-known, when G is compact, they are essentially the same notion. When G is non-compact, this problem arised in 1970's. The first result was obtained by T. Hirai [3] in 1972 which says that they are really different. More precisely, there is a tempered IED which cannot be expressed as a linear combination of irreducible characters. Later, in 1974, Fomin and Shapovalov [1] proved that characters are constant coefficient IEDs (see Definition 1.2). In this article we report that a constant coefficient IED can be written as a linear combination of irreducible characters. Consequently, the space of constant coefficient IEDs and the linear span of irreducible characters with the same eigenvalue are identical (see Theorem 3.2).

§ 1. Constant coefficient IEDs. Let $\mathfrak{g}$ be the Lie algebra of G and $\mathfrak{g}_c$ its complexification. In this paper, we denote a Lie group by Roman