31. *Pluricanonical Maps of Minimal 3-Folds*

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Introduction. In this paper we study pluricanonical maps of non-singular 3-folds of general type over C, provided that they have minimal models. Details will be published elsewhere.

Our main result is stated as follows.

Theorem. *Let X be a minimal 3-fold of general type with index r and let K_X denote the canonical divisor. Then the n-ple pluricanonical map $\Phi_{|nK_X|}$ is birational for $n \geqq n_0$ where*

$$n_0=9 \quad \text{if } r=1,$$
$$n_0=8 \quad \text{if } r=1 \quad \text{and if } X \text{ is } \boldsymbol{Q}\text{-factorial},$$
$$n_0=13 \quad \text{if } r=2,$$
$$n_0=4r+4 \quad \text{if } 3 \leqq r \leqq 5,$$
$$n_0=4r+3 \quad \text{if } r \geqq 6.$$

For the definition of pluricanonical maps, see section 1.

The problem of the birationality of pluricanonical maps for 3-folds has been treated by Benveniste and Matsuki [5] for minimal and non-singular 3-folds. Actually they proved that $\Phi_{|nK_X|}$ is birational for $n \geqq 8$.

When we consider the birationality problem for 3-folds admitting minimal models, we can assume that the 3-folds are minimal. It is conjectured that all 3-folds of general type have minimal models.

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§1. Preliminaries.

Definition 1. Let X be a normal projective variety. A Weil divisor D is said to be $\boldsymbol{Q}$-Cartier if mD is a Cartier divisor for some positive integer m. X is said to be $\boldsymbol{Q}$-factorial if any Weil divisor is $\boldsymbol{Q}$-Cartier. A $\boldsymbol{Q}$-Cartier divisor D is defined to be numerically effective or nef if $(D \cdot C) \geqq 0$ for any irreducible curve C on X.

Definition 2 (Reid [6]). Let X be a normal projective variety, and K_X the canonical divisor. We say that X has only canonical singularities, if K_X is $\boldsymbol{Q}$-Cartier and for a resolution $\mu : \tilde{X} \to X$ there is a natural morphism $\mu^* \omega_X^{[s]} \to \omega_{\tilde{X}}^{\otimes s}$ for any $s \geqq 1$. The minimum integer r such that rK_X is Cartier is called the index of X.

Definition 3 (Reid [6], [7]). Let X be a normal projective variety. X is said to be minimal or a minimal model if X has only canonical singu-