27. *On Removable Singularities of Certain Harmonic Maps*

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1. Statement of result. Let Ω be a domain in $\boldsymbol{R}^n$ and let (M, g) be a Riemannian manifold of dimension m. We assume that M is isometrically embedded in Euclidean space $\boldsymbol{R}^k$. The equation of harmonic maps from Ω into M is given as follows.

$$\Delta u^\alpha(x)=\sum_{i=1}^n A^\alpha_{u(x)}(D_iu(x), D_iu(x)) \qquad \alpha=1, \cdots, k \tag{1.1}$$

where $A_{u(x)}(\cdot, \cdot)$ is the second fundamental form of M at $u(x)$. This is the Euler-Lagrange equation of the energy functional

$$E(u)=\int_\Omega e(u)(x)dx \qquad \text{where } e(u)(x)=|Du(x)|^2. \tag{1.2}$$

(Hereafter, we denote $e(u)(x)$ simply as $e(u)$.)

The purpose of this article is to give a regularity result for a certain class of weak solutions of (1.1). $H^1(\Omega, \boldsymbol{R}^k)$ denotes the Sobolev space of order 1 from Ω to $\boldsymbol{R}^k$. $H^1(\Omega, M)$ is the subset of $H^1(\Omega, \boldsymbol{R}^k)$ consisting of maps having image almost everywhere in M and $L^\infty(\Omega, M)$ is defined similarly.

Definition 1.1 ([8]). A map $u \in H^1(\Omega, M) \cap L^\infty(\Omega, M)$ is called a *stationary map* if the following conditions are satisfied.

(1) For any $\eta \in C_0^\infty(\Omega, \boldsymbol{R}^k)$ we have

$$\int_\Omega \sum_{\alpha=1}^k \sum_{i=1}^n (D_iu^\alpha D_i\eta^\alpha + A^\alpha_u(D_iu, D_iu)\eta^\alpha)dx=0. \tag{1.3}$$

(Then, u is called a *weakly harmonic map*.)

(2) For each one-parameter family $\{F_t\}$ of diffeomorphisms of Ω which are equal to the identity outside a compact set of Ω and with $F_0=\text{id.}$, we have

$$(d/dt)E(u\circ F_t)|_{t=0}=0. \tag{1.4}$$

Remark 1.2. It is known that continuous harmonic maps are smooth stationary maps (see [8]).

The main result is as follows.

Theorem 1.3. *Let B be the unit ball in $\boldsymbol{R}^n$ $(n\geqq 3)$ with the center at the origin and let (M, g) be a Riemannian manifold of dimension m. Let $u \in H^1(B, M) \cap L^\infty(B, M)$ be a stationary map. Suppose that u is of class C^2 in $B-\{0\}$ and the integral $\int_B |Du|^n dx$ is finite. Then, u is extended as a smooth harmonic map from B to M.*

Remark 1.4. (1) In case $n=2$, isolated singular points are removable for each weakly harmonic map ([7, Theorem 3.6]).