105. *Family of Jacobian Manifolds and Characteristic Classes of Surface Bundles*

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1. Introduction. In our previous papers ([2], [3], [4]), we have defined characteristic classes for *surface bundles*, namely differentiable fibre bundles whose fibres are closed orientable surfaces, and investigated general properties of them. The purpose of the present note is to announce new results concerning them. More precisely we study cohomological properties of the natural map from a given surface bundle to its associated "family of Jacobian manifolds" and by using them, we derive a rather strong linear dependence relations among our characteristic classes.

2. Review of the definition of characteristic classes. We begin by recalling the definition of our characteristic classes of surface bundles very briefly (see [3] for details). Let $\pi: E \to X$ be an oriented surface bundle with fibre Σ_g=closed orientable surface of genus $g \geqq 2$ and let ξ be the tangent bundle of π, namely it is the subbundle of the tangent bundle of E consisting of vectors which are tangent to the fibres. We denote $e(\xi) \in H^2(E; \boldsymbol{Z})$ for the Euler class of ξ and define a cohomology class $e_i \in H^{2i}(X; \boldsymbol{Z})$ by $e_i = \pi_*(e^{i+1}(\xi))$ where $\pi_*: H^{2(i+1)}(E; \boldsymbol{Z}) \to H^{2i}(X; \boldsymbol{Z})$ is the Gysin homomorphism. Next let us choose a fibre metric on ξ so that each fibre $E_x = \pi^{-1}(x)$ $(x \in X)$ inherits a Riemannian metric. Now let η be the vector bundle over X whose fibre over $x \in X$ is $H^1(E_x; \boldsymbol{R})$. If we identify $H^1(E_x; \boldsymbol{R})$ with the space of harmonic 1-forms on E_x, then the $*$-operator on $H^1(E_x; \boldsymbol{R})$ satisfies $*^2 = -1$. Hence it induces a structure of complex g-dimensional vector bundle on η. Let $c_i \in H^{2i}(X; \boldsymbol{Z})$ be its Chern class.

The above definition can be made at the universal space level. Namely if we denote $\mathcal{M}_g$ and $\mathcal{M}_{g,*}$ respectively for the mapping class groups of Σ_g and Σ_g relative to the base point $* \in \Sigma_g$, then the natural exact sequence

$$1 \longrightarrow \pi_1(\Sigma_g) \longrightarrow \mathcal{M}_{g,*} \xrightarrow{\pi} \mathcal{M}_g \longrightarrow 1$$

serves as the universal Σ_g-bundle and we have the universal cohomology classes

$$e \in H^2(\mathcal{M}_{g,*}; \boldsymbol{Z})$$
$$e_i = \pi_*(e^{i+1}), \qquad c_i \in H^{2i}(\mathcal{M}_g; \boldsymbol{Z}).$$