96. On Differential Operators and Congruences for Siegel Modular Forms of Degree Two

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§1. Introduction. We study congruences between Siegel modular forms of degree two and different weight by using differential operators. In the degree one case, such congruences were studied by Serre [6] and Swinnerton-Dyer [8]. For the degree two case, we refer to Kurokawa [2]. We denote by $M_k(\Gamma_n)$ (resp. $M_k^\infty(\Gamma_n)$, $S_k(\Gamma_n)$) the $\boldsymbol{C}$-vector space of holomorphic Siegel modular forms (resp. C^∞-modular forms, holomorphic cusp forms) of degree n and weight k. For a subring R of $\boldsymbol{C}$, we denote by $M_k(\Gamma_n)_R$ the R-submodule of $M_k(\Gamma_n)$ consisting of Siegel modular forms which have Fourier coefficients in R. This paper is an abstract of [5].

§2. General results. We introduce certain differential operators. For a variable $Z=\begin{pmatrix} z_1 & z_3 \\ z_3 & z_2 \end{pmatrix}$ on H_2 of Siegel upper half plane of degree two, we put

$$Y=\frac{1}{2i}(Z-\bar{Z})=\begin{pmatrix} y_1 & y_3 \\ y_3 & y_2 \end{pmatrix}, \qquad \frac{d}{dZ}=\begin{pmatrix} \dfrac{\partial}{\partial z_1} & \dfrac{1}{2}\cdot\dfrac{\partial}{\partial z_3} \\ \dfrac{1}{2}\cdot\dfrac{\partial}{\partial z_3} & \dfrac{\partial}{\partial z_2} \end{pmatrix}$$

and $dY=dy_1dy_2dy_3$. For integers k and $r\geqq 0$, we define a differential operator δ_k acting on a C^∞-function f on H_2 by

$$\delta_k f=|Y|^{-k+(1/2)}\left|\frac{d}{dZ}\right|(|Y|^{k-(1/2)}f)$$

and put $\delta_k^r=\delta_{k+2r-2}\cdots\delta_{k+2}\delta_k$. We understand that δ_k^0 is the identity operator. These differential operators were studied by Maass [4]. By Harris [1, 1.5.3], δ_k^r maps $M_k^\infty(\Gamma_2)$ to $M_{k+2r}^\infty(\Gamma_2)$.

Next, we make a survey of a holomorphic projection. We set $V=\{Y\in M(2,\boldsymbol{R})\,|\,Y>0\}$. For $f\in M_w^\infty(\Gamma_2)$, let $f(Z)=\sum_T a(T,Y,f)q^T$ be its Fourier expansion, where $q^T=\exp(2\pi i\,\mathrm{Tr}\,(TZ))$ and T runs over all half-integral matrices of size two. We put

$$P_w(f)=\sum_{T>0}P(w,T,a(T,Y,f))q^T,$$

where

$$P(w,T,a(T,Y,f))=\frac{\int_V a(T,Y,f)e^{-4\pi\,\mathrm{Tr}\,(TY)}|Y|^{w-3}dY}{\int_V e^{-4\pi\,\mathrm{Tr}\,(TY)}|Y|^{w-3}dY}$$