86. A Stability Theorem on the Boundary Identification for Coefficients of Hyperbolic Equations

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In this note, we consider the hyperbolic equation

(1) $\partial_t^2 u+(-\partial_x^2+p(x))u=0 \qquad (0<x<1,\ -\infty<t<\infty)$

with the boundary condition

(2) $(-\partial_x+h)u|_{x=0}=(\partial_x+H)u|_{x=1}=0 \qquad (-\infty<t<\infty)$

and with the initial condition

(3) $u|_{t=0}=a_0(x),\quad \partial_t u|_{t=0}=a_1(x) \quad (0<x<1).$

We suppose that the coefficients $P\equiv(p, h, H)\in C_x^0[0,1]\times \boldsymbol{R}\times \boldsymbol{R}$ and the initial values $a\equiv(a_0, a_1)\in H_x^1(0,1)\times L_x^2(0,1)$ are unknown, while the boundary values of the solution $u=u(x,t)\in C_t^0((-\infty,\infty)\to H_x^1(0,1))\cap C_t^1((-\infty,\infty)\to L_x^2(0,1))\subset C_{x,t}^0((-\infty,\infty)\times[0,1])$

(4) $u|_{x=0}=f_0(t) \quad u|_{x=1}=f_1(t) \quad (-T\leqq t\leqq T)$

are observed and known for some $T>0$. In order to study the identifiability (see [2], e.g.), let us consider the model equation

(5) $\partial_t^2 v+(-\partial_x^2+q(x))v=0 \qquad (0<x<1,\ -\infty<t<\infty)$

with

(6) $(-\partial_x+j)v|_{x=0}=(\partial_x+J)v|_{x=1}=0 \qquad (-\infty<t<\infty)$

and

(7) $v|_{t=0}=b_0(x),\quad \partial_t v|_{t=0}=b_1(x) \qquad (0<x<1)$

for $Q\equiv(q, j, J)\in C_x^0[0,1]\times\boldsymbol{R}\times\boldsymbol{R}$ and $b\equiv(b_0, b_1)\in H_x^1(0,1)\times L_x^2(0,1)$. Then, the functions

(8) $\varepsilon_0(t)=g_0(t)-f_0(t),\quad \varepsilon_1(t)=g_1(t)-f_1(t) \quad (-T\leqq t\leqq T)$

stand for the errors of identification, where

(9) $g_0(t)=v|_{x=0},\quad g_1(t)=v|_{x=1} \quad (-T\leqq t\leqq T).$

To state our results, we introduce the following:

Notation 1. $A_{p,h,H}$ denotes the Sturm-Liouville operator $-\partial_x^2+p(x)$ in $L_x^2(0,1)$ with the boundary condition

$$(-\partial_x+h)\cdot|_{x=0}=(\partial_x+H)\cdot|_{x=1}=0.$$

Notation 2. $\sigma(A_{p,h,H})=\{\lambda_n\}_{n=0}^\infty$ $(-\infty<\lambda_0<\lambda_1<\cdots\to\infty)$ and $\{\phi\}_{n=0}^\infty$ denote the eigenvalues and the eigenfunctions of $A_{p,h,H}$, respectively, the latter being normalized by $\|\phi_n\|_{L_x^2(0,1)}=1$.

Notation 3. The equation (1) with (2)–(3) is denoted by $E(P, a)$.

Definition 1. We say $E(P, a)\in G$ if

(10) $(a_n^0)^2+(a_n^1)^2\neq 0$

for any $n\in \boldsymbol{N}\equiv\{0, 1, 2, \cdots\}$, where