67. *Explosion Problems for Symmetric Diffusion Processes*

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(Communicated by Kôsaku YOSIDA, M. J. A., Sept. 12, 1984)

§ 1. **Introduction.** Let L be a strictly elliptic partial differential operator with measurable coefficients of the form:

$$L=\frac{1}{b}\sum_{i,j=1}^{n}\frac{\partial}{\partial x_i}\left(a_{ij}\frac{\partial}{\partial x_j}\right)$$

where (a_{ij}) is symmetric and $b>0$. We assume that for each non-empty compact subset K of $\boldsymbol{R}^n$, there exists a constant $\lambda=\lambda(K)$ such that

$$\lambda^{-1}|\xi|^2\leqq\sum_{i,j=1}^{n}a_{ij}(x)\xi_i\xi_j\leqq\lambda|\xi|^2$$

and

$$b(x)\leqq\lambda$$

for all x in K and ξ in $\boldsymbol{R}^n$. Then we can construct a unique minimal diffusion process $(X_t, \zeta, P_x)_{x\in\boldsymbol{R}^n}$ by using the theory of Dirichlet spaces, Fukushima [2] (see also Morrey [5]) where ζ is the explosion time of the process, i.e. $\lim_{t\nearrow\zeta(\omega)}|X_t(\omega)|=+\infty$ if $\zeta(\omega)<+\infty$. One of the basic problems for the diffusion processes is to find conditions for conservativeness and explosion. Such conditions for one dimensional diffusion processes have been established by Feller [1] in connection with the classification of boundary points. His conditions are given in terms of the scale and speed measures. In multidimensional cases, Hasminskii [3] has obtained sufficient conditions for conservativeness and explosion for diffusion processes which can be constructed by means of Itô's stochastic differential equations. Hasminskii's idea (see McKean [4]) can not be applied to our cases since the coefficients a_{ij} of the above operator L are not necessarily smooth. However we can use the theory of Dirichlet spaces to get conditions for conservativeness and explosion.

§ 2. **α-equilibrium potential and α-capacity** (Fukushima [2]). Let $\boldsymbol{B}_n$ be the closed unit ball $\{|x|\leqq 1\}$ in $\boldsymbol{R}^n$ and τ_0 the first hitting time of $\boldsymbol{B}_n$ by the process X_t. The α-equilibrium potential $e_\alpha(x)$ of $\boldsymbol{B}_n$ $(\alpha\geqq 0)$ is defined by

$$e_\alpha(x)=\begin{cases}E_x[e^{-\alpha\tau_0}] & \text{for } \alpha>0\\ P_x[\tau_0<\zeta] & \text{for } \alpha=0.\end{cases}$$

For u, v in the space $C_0^\infty(\boldsymbol{B}^n)$ of infinitely differentiable, real valued