26. An Application of the Perturbation Theorem for m-Accretive Operators

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The present note is concerned with the semi-linear equation $-\Delta u(x)+\beta(x,u(x))=f(x)$ on the whole space $\boldsymbol{R}^n$. This equation was recently treated by Sohr [4]. As an application of the perturbation theorem in Okazawa [3] we shall improve the result obtained in [4]. Here, it should be noted that a quite general theorem has been established by Brezis-Crandall-Pazy [1] and Konishi [2] if β does not explicitly depend on x: $\beta(x,u)=\beta(u)$.

1. Preliminaries. We consider only real-valued functions. An operator A (with domain $D(A)$ and range $R(A)$) in $L^2=L^2(\boldsymbol{R}^n)$ is said to be *accretive* (or monotone) if

$$(Au-Av,\ u-v)\geqq 0 \qquad \text{for } u,\ v\in D(A).$$

We say that an accretive operator A is *m-accretive* if $R(1+\lambda A)=L^2$ for some (and hence for every) $\lambda>0$. The *Yosida approximation* $\{B_\varepsilon\}$ of an m-accretive operator B is defined by

$$B_\varepsilon=\varepsilon^{-1}[1-(1+\varepsilon B)^{-1}], \qquad \varepsilon>0.$$

The following lemma is a Hilbert space case of Lemma 6.2 in [3].

Lemma 1. *Let A and B be m-accretive operators in L^2, with $D(A)\cap D(B)$ non-empty. Assume that there exist a constant b $(0\leqq b<1)$ and a nondecreasing function $\psi(r)\geqq 0$ of $r\geqq 0$ such that for all $u\in D(A)$ and $\varepsilon>0$,*

$$(Au,B_\varepsilon u)\geqq -\psi(\|u\|)-b\,\|B_\varepsilon u\|^2.$$

Then $A+B$ is also m-accretive in L^2.

Now, let J be an open interval on $\boldsymbol{R}$ and β be a real-valued function of class $C^1(\boldsymbol{R}^n\times J)$:

$$\beta(x,s)=\beta(x_1,x_2,\cdots,x_n,s).$$

We assume that $0\in J$ and

(i) $\beta(x,0)=0$ for every $x\in\boldsymbol{R}^n$, and $\partial\beta/\partial s\geqq 0$ on $\boldsymbol{R}^n\times J$.

Then we can introduce the following accretive operator $\tilde\beta$ in L^2:

$$D(\tilde\beta)=\{u\in L^2;\ u(x)\in J(\text{a.e.}),\ \beta(x,u(x))\in L^2\},$$
$$\tilde\beta u(x)=\beta(x,u(x)) \qquad \text{for } u\in D(\tilde\beta).$$

Lemma 2. *Let $\tilde\beta$ be the accretive operator as above. Then $\tilde\beta$ is m-accretive if*

(ii) *for every $x\in\boldsymbol{R}^n$, $\beta(x,\cdot): J\to\boldsymbol{R}$ is onto.*

Proof. Since $\tilde\beta$ is closed, it suffices to show that $R(\tilde\beta+1)$ contains