24. A Compact-Like Space which does not have a Countable Cover by C-Scattered Closed Subsets

By Tsugunori NOGURA
Department of Mathematics, Ehime University

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Let K denote a class of spaces which are hereditary with respect to closed subspaces. Let FK denote the class of all $X=\bigcup\{X_m : m\leq n\}$, where X_m is a closed subset of X for each $m\leq n$, $n\in N$ (N denotes the natural numbers) and $X_m\in K$. Let C denote the class of all compact spaces. Then $FC=C$.

The topological game $G(K,X)$ is introduced and studied by R. Telgársky ([1], [2]). We use the notations in [1]. Each space considered here is assumed to be completely regular.

The following theorems are proved by R. Telgársky:

(a) ([1], Theorem 11.1). Let X be a hereditarily paracompact K-like ([1], p. 195) space. Then $X=\bigcup\{X_n : n\in N\}$, where X_n is a closed FK-scattered subset of X for each $n\in N$.

(b) ([2], Theorem 1.3). Let X be a K-like space. Then $X=\bigcup\{X_n : n\in N\}$, where X_n is a K-scattered subset of X for each $n\in N$.

(c) ([2], Remark 1.5). Let X be a K-like space. Assume that each open subset of X is the union of a σ-locally finite family of closed sets (in particular, that X is totally normal or hereditarily paracompact), then $X=\bigcup\{X_n : n\in N\}$, where X_n is a closed FK-scattered subset of X for each $n\in N$.

The following problem is posed by R. Telgársky ([2], Remark 1.5):

Does each K-like space have a countable cover by K-scattered closed subset?

The following simple example gives a negative answer to the above problem.

Theorem. (CH) *There exists a compact-like space X which does not have a countable cover by C-scattered closed subsets.*

Proof. Let $I=[0,1]$ be the closed unit interval. Well-order $I=\{x_\alpha : \alpha<\omega_1\}$, where ω_1 is the first uncountable ordinal number. Let $[0,\omega_1)$ ($[0,\omega_1]$) be the space of ordinal numbers less than (less than or equal to) ω_1 with the interval topology. For each $\alpha<\omega_1$, put $M_\alpha=\{(\alpha, x_\beta)\in[0,\omega_1)\times I,\ \beta\leq\alpha\}$ and $X=\bigcup\{M_\alpha : \alpha<\omega_1\}\cup\{\omega_1\}\times I$. We will show that the subspace X of the space $[0,\omega_1]\times I$ has desired properties. First we show that X is compact-like. Put $E_0=X$, $E_1=\{\omega_1\}\times I$. Let