18. On Nonlinear Hyperbolic Evolution Equations with Unilateral Conditions Dependent on Time

By Shigeru SASAKI

(Communicated by Kôsaku YOSIDA, M. J. A., Feb. 12, 1983)

1. Introduction. In this paper we are concerned with the strong solution of the following nonlinear hyperbolic evolution equation

$$\text{(E)} \qquad \frac{d^2u}{dt^2}(t)+Au(t)+\partial I_{K(t)}\left(\frac{du}{dt}(t)\right)\ni f(t), \qquad 0\leqq t\leqq T$$

in a real Hilbert space H. Here A is a positive self-adjoint operator in H. For each $t\in[0,T]$, $K(t)$ is a closed convex subset of H and $\partial I_{K(t)}$ is the subdifferential of $I_{K(t)}$ which is the indicator function of $K(t)$. We denote the inner product and the norm in H by $(\cdot,\cdot)$ and $|\cdot|$, respectively. For each $t\in[0,T]$, let $P(t)$ denote the projection operator of H onto $K(t)$. Moreover we assume the following conditions for A and $K(t)$.

(A.1) There exists $a\in L^2(0,T;H)$ such that for a.e. $t\in[0,T]$, every $x\in K(t)$ and $\varepsilon>0$, $(1+\varepsilon A)^{-1}(x+\varepsilon a(t))\in K(t)$.

(A.2) There exists a strongly absolutely continuous function $b:[0,T]\to H$ such that $b(t)\in D(A^{1/2})\cap K(t)$ for a.e. $t\in[0,T]$ and $A^{1/2}b\in L^1(0,T;H)$.

(A.3) For each $x\in H$, $P(\cdot)x:[0,T]\to H$ is strongly measurable.

(A.4) There exists a continuous function $\omega:R^+\to R^+$ such that for each $h\in\,]0,T[$ and $v\in C([0,T];H)$,

$$\int_0^{T-h}|P(s+h)v(s)-P(s)v(s)|^2\,ds\leqq h^2\omega\left(\sup_{t\in[0,T]}|v(t)|\right).$$

Definition. Let $u:[0,T]\to H$. Then u is called a strong solution of (E) on $[0,T]$ if (i) $u\in C^1([0,T];H)$, (ii) du/dt is strongly absolutely continuous on $[0,T]$, (iii) $u(t)\in D(A)$ and $du(t)/dt\in K(t)$ for a.e. $t\in[0,T]$ and (iv) u satisfies (E) for a.e. $t\in[0,T]$.

Now we state our main theorem.

Theorem. *Suppose that the assumptions stated above are satisfied. Then for each $f\in W^{1,2}(0,T;H)$, $u_0\in D(A)$ and $v_0\in D(A^{1/2})\cap K(0)$, the equation* (E) *has a unique strong solution u on $[0,T]$ with $u(0)=u_0$ and $(du/dt)(0)=v_0$. Moreover, u has the following properties.*

(i) $Au\in L^\infty(0,T;H)$.

(ii) $u(t)\in D(A^{1/2})$ *for every* $t\in[0,T]$ *and* $A^{1/2}u\in C([0,T];H)$.

(iii) $du(t)/dt\in D(A^{1/2})$ *for* a.e. $t\in[0,T]$ *and* $A^{1/2}du/dt\in L^\infty(0,T;H)$.

(vi) $d^2u/dt^2\in L^2(0,T;H)$.

In the case where $K(t)=K$ is independent of t, the existence and