101. *Microlocal Study of Sheaves. II*

Constructible Sheaves

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Introduction. On a real (resp. complex) analytic manifold X, we prove that a complex of sheaves is constructible if and only if it satisfies some finiteness property and if its micro-support [5] is a subanalytic (resp. complex analytic) Lagrangian set. Thus we may study the functorial properties, including contact transformations [6], with our previous results on the micro-support of sheaves. As an application we give a direct image theorem for regular holonomic modules in the non proper case.

1. Let X be a real analytic manifold. We use the same notations as in [6]. In particular $SS(F)$ is the micro-support in T^*X of a complex of sheaves on X. In this note we shall only consider sheaves of vector spaces, in order to simplify the discussion.

Let F be a complex of sheaves on X. We shall say that F is weakly $\boldsymbol{R}$-constructible if there exists a subanalytic stratification such that the restriction of the cohomology groups of F to each stratum is locally constant. We denote by $D^+(X)$ the derived category of complexes of sheaves bounded from below and by $D^+_{w\boldsymbol{R}c}(X)$ the full subcategory consisting of weakly $\boldsymbol{R}$-constructible complexes.

Recall (cf. [2]) that a complex $F \in Ob(D^b(X))$ is said to be $\boldsymbol{R}$-constructible if $F \in Ob(D^+_{w\boldsymbol{R}c}(X))$ and moreover for all $x \in X$, the space $H^j(F)_x$ is finite-dimensional. We denote by $D^b_{\boldsymbol{R}-c}(X)$ the full subcategory of $D^+_{w\boldsymbol{R}c}(X)$ of $\boldsymbol{R}$-constructible complexes.

Theorem 1.1. *Let $F \in Ob(D^+(X))$. The following conditions are equivalent.*

i) $F \in Ob(D^+_{w\boldsymbol{R}c}(X))$.

ii) *$SS(F)$ is contained in a subanalytic and isotropic set of T^*X* (isotropic: There exists a dense open smooth manifold in $SS(F)$ on which the fundamental 1-form vanishes).

iii) *$SS(F)$ is a closed conic Lagrangian subanalytic set of T^*X.*

For the proof we use the technics of [1] and [5], [6].

As a corollary of Theorem 1.1 we prove that if Y is a submanifold of X and $F \in Ob(D^b_{\boldsymbol{R}c}(X))$ then $\nu_Y(F)$ the specialization of F along Y

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