82. *Algebraic Geometry of Soliton Equations**)

By Motohico MULASE
Mathematical Sciences Research Institute, 2223 Fulton Street, Berkeley, CA 94720

(Communicated by Heisuke HIRONAKA, M. J. A., June 14, 1983)

The purpose of this paper is to classify all the subdynamical systems of the $K-P$ dynamical system $(\hat{G}, T)$ defined in [2] in terms of commutative algebras. We show that every orbit in $(\hat{G}, T)$ is locally isomorphic to a certain first cohomology group $H^1(A)$ associated with a commutative algebra A and the $K-P$ dynamical system is nothing but a dynamical system of a linear motion on this cohomology group. In the case of so called quasi-periodic solutions, it is known that the $K-P$ dynamical system determines a linear motion on the Jacobian varieties of algebraic curves. Our results are the widest extension of this classical result. We also characterize all the finite dimensional orbits in $(\hat{G}, T)$. We show that an orbit is of finite dimension if and only if our cohomology group $H^1(A)$ is isomorphic to $H^1(C, \mathcal{O}_C)$ for a certain complete algebraic curve C defined over the complex number field $\boldsymbol{C}$. This enables us to solve the *Schottky problem* in the following manner; an Abelian variety is a Jacobian variety if and only if it appears as an orbit in $(\hat{G}, T)$ (cf. [4]).

In this paper we use notations defined in [1] and [2] freely.

1. Subdynamical systems of $(\hat{G}, T)$ and commutative algebras. Let $H=\boldsymbol{C}((\partial^{-1}))$. This is a maximal commutative subalgebra in the Lie algebra E of [1]. Let $\mathcal{A}=\{A\subset H \mid A$ is a $\boldsymbol{C}$-subalgebra with unity and $A\cap \boldsymbol{C}[[\partial^{-1}]]\cdot\partial^{-1}=0\}$. Define $X_A=\{S\in G \mid SAS^{-1}\subset D\}$ and $\hat{X}_A=\{S\partial S^{-1} \mid S\in X_A\}$. The condition $A\cap \boldsymbol{C}[[\partial^{-1}]]\cdot\partial^{-1}=0$ intends to avoid the trivial case $X_A=\phi$. Also by this condition A has transcendence degree 1 over $\boldsymbol{C}$. Mikio Sato has originally introduced the notion of A to study several orbits.

Proposition 1.1. *X_A is a time invariant subspace in G. So $(\hat{X}_A, T)$ is a subdynamical system of $(\hat{G}, T)$.*

Proof. For every $S\in X_A$ we have a unique solution $S(t)$ to the Sato equation starts at $S(0)=S$ ([1]). So it is sufficient to prove $\partial/\partial t_n(S(t)AS(t)^{-1})\subset D$ for every $n\geqslant 1$. Define

$$L=S(t)\partial S(t)^{-1},\quad Z=\sum_{n=1}^{\infty}(L^n)_+dt_n \quad\text{and}\quad Z^c=-\sum_{n=1}^{\infty}(L^n)_-dt_n.$$

*) Supported by the Harvard Committee on the Educational Project for Japanese Mathematical Scientists.