80. *On Degrees of Non-Roughness of Real Projective Varieties*

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Related to *the Hilbert's 16th problem*, the following problem is presented since 1965 (see Gudkov [3] p. 485, [4] p. 6 and Risler [6] p. 23):

Problem. *Does each real plane algebraic curve of a fixed order have a well-defined and finite degree of non-roughness?*

In short, the degree of non-roughness of a curve (or a variety) represents its topological degeneration (cf. Definition 2).

The purpose of the present note is to answer this problem *affiermatively* thanks to the stratification theory of R. Thom ([8]). Further we see that *the degrees of non-roughness of real projective varieties of a fixed order are well-defined and have a finite upper bound* (Theorem 1).

We consider the "equivariant" isotopy type of a complexified variety: Theorem 2 (cf. [7]).

1. Formulations of results. Let $\boldsymbol{R}P^{N_1}\times\cdots\times\boldsymbol{R}P^{N_s}$ be the set of $f=(f_1, \cdots, f_s)$ considered modulo non-zero-constants in each component, where f_i is a non-zero homogeneous polynomial of order d_i, with variables $x_0, x_1, \cdots, x_n$ and with coefficients in $\boldsymbol{R}$, and $N_i=\binom{n+d_i}{n}-1$ $(i=1, \cdots, s)$.

We mean by a *real projective variety* of order $(d_1, \cdots, d_s)$ simply a point of $\boldsymbol{R}P^{N_1}\times\cdots\times\boldsymbol{R}P^{N_s}$. Each real projective variety $[f]$ determines naturally a subset $V[f]$ of $\boldsymbol{R}P^n$ and invariant subset $\boldsymbol{C}V[f]$ of $\boldsymbol{C}P^n$ under the complex conjugation, by the equation $f_1(x)=\cdots=f_s(x)=0$.

The first half of the sixteenth problem of Hilbert is regarded, in an extended sense, as the investigation of isotopy types of pairs $(\boldsymbol{R}P^n, V[f])$ (cf. [4]).

Let $\mathcal{A}$ (resp. $\mathcal{B}$) be a semi-algebraic stratification of a closed subset A of $\boldsymbol{R}P^n$ (resp. B of $\boldsymbol{C}P^n$, $\mathcal{B}$ being invariant under the complex conjugation $\boldsymbol{C}P^n\to\boldsymbol{C}P^n$). (A subset of an algebraic manifold is *semi-algebraic* if it is semi-algebraic on each affine chart.)

Definition 1. Two real projective varieties $[f], [f']\in\boldsymbol{R}P^{N_1}\times\cdots\times\boldsymbol{R}P^{N_s}$ of a same degree $(d_1, \cdots, d_s)$ are called *isotopic rel.* $\mathcal{A}$ (resp.