73. *Table of the Fourier Coefficients of Eisenstein Series of Degree 3*

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§1. Introduction. Let $\mathcal{H}_n$ be the Siegel upper-half space of degree n, and $\boldsymbol{Z}$ the variable on $\mathcal{H}_n$, then Eisenstein series $\psi_k(\boldsymbol{Z})$ of degree n and weight k is defined by

$$\psi_k(\boldsymbol{Z})=\sum|\boldsymbol{CZ}+\boldsymbol{D}|^{-k},$$

where $\boldsymbol{C}$ and $\boldsymbol{D}$ run over complete representatives of the equivalent classes of $n\times n$ square, coprime and symmetric pairs of integral matrices. $\psi_k(\boldsymbol{Z})$ can be expanded to the Fourier series:

$$\psi_k(\boldsymbol{Z})=\sum_{\boldsymbol{T}\geqq 0} a_k(\boldsymbol{T})\,e^{2\pi i\sigma(\boldsymbol{TZ})}.$$

In [2], we have given explicit formulas for the Fourier coefficients $a_k(\boldsymbol{T})$ of $\psi_k(\boldsymbol{Z})$ in the case when $n=3$ and T are positive definite semi-integral primitive ternary matrices. By means of these formulas, we have calculated the numerical values of $a_k(\boldsymbol{T})$ for $k\leqq 24$ and for "smaller" T's. In the following, we give a table of such values which will be useful for the arithmetical investigations. (For technical reasons, this gives only a part of our result whose scope will be specified below. We are willing to communicate our result to anyone interested.)

§2. Organization and usage of the table. Concerning the positive definite semi-integral primitive ternary symmetric matrices we can utilize the table by Brandt-Intrau [1]. In their table the matrix

$$\begin{pmatrix} a_1 & a_2 & a_3 \\ a_{23} & a_{13} & a_{12} \end{pmatrix},$$

is used to indicate the ternary quadratic form

$$f=a_1x_1^2+a_2x_2^2+a_3x_3^2+a_{12}x_1x_2+a_{13}x_1x_3+a_{23}x_2x_3.$$

From this we obtain a matrix $\boldsymbol{T}_f=(b_{ij})$, where $b_{ii}=a_i$ $1\leqq i\leqq 3$ and $b_{ij}=b_{ji}=a_{ij}/2$ $1\leqq i<j\leqq 3$. If f is a positive definite ternary quadratic form with integer coefficients, then $\boldsymbol{T}_f$ is a positive definite semi-integral ternary matrix. In [1] d means $(-1)/2\times\det(2\boldsymbol{T}_f)$. However we prefer to use $d=d_f$ to denote $\det(2\boldsymbol{T}_f)/2$ which is positive. Since by [2] we know that the Fourier coefficient $a_k(\boldsymbol{T})$ of Eisenstein series of degree 3 is a genus invariant, we employ as the repesentative of each genus the first form given in [1]. Our table consists of Tables I and II.