71. Classification of Logarithmic Fano 3-Folds

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§1. **Introduction.** The purpose of this note is to outline our recent results on the structure of logarithmic Fano 3-folds. Details will be published elsewhere. Our proof of the results is based on the theory of threefolds whose canonical bundles are not numerically effective, due to S. Mori [6], and the theory of open algebraic varieties, due to S. Iitaka [2].

Let X be a non-singular projective variety over an algebraically closed field k of characteristic zero. Let $D=D_1+D_2+\cdots+D_s$ be a divisor with simple normal crossings on X.

A pair (X, D) is called a logarithmic Fano variety if $-K_X-D$ is an ample divisor, In the case where $D=0$, X turns out to be a Fano variety in the usual sense.

A logarithmic Fano variety of dimension two may be called a logarithmic del Pezzo surface.

§2. **General properties.** Let (X, D) be a logarithmic Fano variety of an arbitrary dimension. By using Norimatsu vanishing theorem [8, Theorem 1], we have the following

Lemma 2.1. (1) *$\kappa(X)=-\infty$ and $\kappa^{-1}(X)=\dim X$.*

(2) Pic $(X)\cong H^2(X, Z)$. *In particular, $\rho(X)=B_2(X)$.*

(3) Pic (X) *is torsion free.*

The boundary D of a logarithmic Fano variety (X, D) satisfies the following

Lemma 2.2. (1) *$D_i \cap D_j \neq \phi$ for any i and j.*

(2) $s \leqq \dim X$.

§3. **Classification of logarithmic del Pezzo surfaces.**

Lemm 3.1. *Let (S, Γ) be a logarithmic del Pezzo surface. Then the Δ-genus* [1, Definition 1.4] *of S with respect to $-K_S-\Gamma$ is as follows:*

(a) *If $\Gamma=0$, then $\Delta(S, -K_S)=1$.*

(b) *If $\Gamma\neq 0$, then $\Delta(S, -K_S-\Gamma)=0$.*

Using the results of T. Fujita [1, pp. 107–110] on polarized varieties of Δ-genera zero, we have the following

Proposition 3.2. *Let (S, Γ) be a logarithmic del Pezzo surface. If $\Gamma\neq 0$, then (S, Γ) is one of the following 7 pairs:*

(i) *$S\cong \boldsymbol{P}^2$, $\Gamma=\Gamma_1$ where Γ_1 is a line.*