67. *Note on the Wiener Compactification and the H^p-Space of Harmonic Functions*

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Introduction. Let R be a hyperbolic Riemann surface. We denote by $HB(R)$ (resp. $HB'(R)$) the class of all bounded harmonic (resp. quasibounded harmonic) functions on R. For p $(1<p<\infty)$, we denote by $H^p(R)$ the class of all harmonic functions u on R such that $|u|^p$ has a harmonic majorant. Then Naim [1] obtained in terms of the Martin boundary that $HB(R)\subset H^p(R)\subset HB'(R)$ for all p. On the other hand the second author [3] proved in terms of Wiener boundary that $\dim HB(R)<\infty$ implies $HB(R)=H^p(R)$ for all p. In this note we shall prove that if any two classes of $HB(R)$, $H^p(R)$ and $HB'(R)$ coincide, then we necessarily have $\dim HB(R)<\infty$. Thus we obtain that $\dim HB(R)<\infty$ if and only if $HB(R)=H^p(R)$ for some p and hence for all p. The first author wishes to express his thanks to Prof. A. Yoshikawa for valuable discussions on L^p-spaces.

1. The H^p-space of harmonic functions. Let R be a hyperbolic Riemann surface and let z_0 be a fixed point in R once for all. Let $\{R_n\}_{n=1}^{\infty}$ be a regular exhaustion of R such that z_0 is contained in all R_n's. We denote by $\mu_n=\mu_{z_0}^{R_n}$ the harmonic measure on the boundary ∂R_n. Note that $\int_{\partial R_n} d\mu_n=1$ for all n.

Definition. A harmonic function u on R belongs to $H^p(R)$, $1\leqq p<\infty$, if and only if the p-mean values

$$\|u\|_{p,n}=\left(\int_{\partial R_n}|u|^p d\mu_n\right)^{1/p}$$

are uniformly bounded in n. Set $H^\infty(R)=HB(R)$, the space of all bounded harmonic functions on R.

Theorem 1 (Naim [1]). (i) $u\in H^p(R)$, $1\leqq p<\infty$, *if and only if* $|u|^p$ *has a harmonic majorant.*

(ii) $HB(R)\subset H^p(R)\subset HB'(R)$, $1<p<\infty$.

2. Lemma on L^p-space. Let X be a compact Hausdorff space and μ be a positive (Radon) measure on X. We denote by $S\mu$ the support of μ. We note that $x\in X$ belongs to $S\mu$ if and only if $\mu(V)>0$ for any open neighborhood V of x. We denote by $L^p=L^p(X,\mu)$ the

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