34. An Example of a Complex of Linear Differential Operators of Infinite Order

By Takahiro KAWAI
Research Institute for Mathematical Sciences, Kyoto University

(Communicated by Shokichi IYANAGA, M. J. A., March 12, 1983)

The purpose of this note is to show a finiteness theorem for a complex of linear differential operators of infinite order acting on the sheaf of holomorphic functions. The complex to be studied arises in the study of ϑ-zerovalue ([6]), and a detailed study of it is an important subject for the further development of [6]. A general result for microfunction solutions will be given in [7].

Let t denote a coordinate system on $\boldsymbol{C}$ and let P and Q respectively denote the matrix of linear differential operators given below:

$$P=\begin{bmatrix} & t \\ 4\pi\sqrt{-1}\left(t\frac{d}{dt}+\frac{1}{2}\right) & \end{bmatrix}$$

$$Q=\begin{bmatrix} & 1 \\ 4\pi\sqrt{-1}\frac{d}{dt} & \end{bmatrix}.$$

If we define Φ and Ψ by $\exp P-1$ $(=\sum_{n=1}^{\infty}P^n/n!)$ and $\exp Q-1$ $(\sum_{n=1}^{\infty}Q^n/n!)$ respectively, then we find ([6])

(1) Φ and Ψ are linear differential operators of infinite order

and

(2) $$\Phi\Psi=\Psi\Phi.$$

For an open subset Ω of $\boldsymbol{C}$, we denote by $K(\Omega)^{\boldsymbol{\cdot}}$ the complex

(3) $$0\longrightarrow\mathcal{O}(\Omega)^2\xrightarrow{(\Phi,\Psi)}\mathcal{O}(\Omega)^4\xrightarrow{\begin{pmatrix}-\Psi\\ \Phi\end{pmatrix}}\mathcal{O}(\Omega)^2\longrightarrow 0$$

determined by Φ and Ψ, where $\mathcal{O}(\Omega)$ denotes the space of holomorphic functions defined on Ω. Let $H^j(K(\Omega)^{\boldsymbol{\cdot}})$ denote its j-th cohomology group. Then we have the following

Theorem. *Let $\Omega(c)$ denote $\{t\in\boldsymbol{C};\ \operatorname{Im}t>c\}$. Then*

(i) $H^0(K(\Omega(c))^{\boldsymbol{\cdot}})\cong\begin{cases}\boldsymbol{C} & \text{for } c\geqq 0\\ 0 & \text{for } c<0\end{cases}$

(ii) $H^1(K(\Omega(c))^{\boldsymbol{\cdot}})\cong\begin{cases}0 & \text{for } c\geqq 0\\ \boldsymbol{C} & \text{for } c<0\end{cases}$

(iii) $H^2(K(\Omega(c))^{\boldsymbol{\cdot}})=0$ *for any* c.

Proof. Since Ψ is with constant coefficients, $\Psi:\mathcal{O}(\Omega)^2\to\mathcal{O}(\Omega)^2$ is surjective for any convex open subset Ω of $\boldsymbol{C}$ ([4]). Hence (iii) is obvi-