30. On 4-Manifolds Fibered by Tori. II

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This is a sequel to our previous note [2]. We will prove a signature formula for torus fibrations (Theorem 6) by combining Novikov additivity and W. Meyer's theorem [4]. This formula seems useful, especially in the study of singular fibers. Some computations will be presented. Also we will give a necessary condition for the existence of good torus fibrations in the sense of [3].

§ 5. The signature formula. Throughout the note, all manifolds will be compact, oriented and smooth. Sign (M) will denote the signature of the homological intersection form $H_2(M;Z)\times H_2(M;Z)\to Z$, where M is a connected 4-manifold with or without boundary.

Let F_i be a singular fiber of a torus fibration $f_i: M_i^4\to B_i^2$, for each $i=1,2$. Let $\{p_i\}=f_i(F_i)$.

Definition. F_1 and F_2 are said to be *topologically equivalent* if there exist neighborhoods U_1, U_2 of p_1, p_2 in $\mathrm{Int}(B_1^2)$, $\mathrm{Int}(B_2^2)$, respectively, and orientation preserving homeomorphisms $h: U_1\to U_2$ and $H: f_1^{-1}(U_1)\to f_2^{-1}(U_2)$, so that (i) $h(p_1)=p_2$ and (ii) $h\circ f_1=f_2\circ H$.

Let $\mathcal{S}$ denote the totality of topological equivalence classes of singular fibers. Let $(1/3)Z=\{m/3\,|\,m\in Z\}\subset \boldsymbol{Q}$.

Theorem 6. *There exists a (practically computable) function* σ: $\mathcal{S}\to(1/3)Z$ *with the following property: If* $\{F_1,\cdots,F_r\}$ *is the set of all the singular fibers of a given torus fibration* $f: M^4\to B^2$ *with* M^4 *closed, then* Sign $(M)=\sum_{i=1}^r \sigma(F_i)$ *holds.*

Consider a situation in which a singular fiber F_0 splits into several singular fibers $F_1',\cdots,F_r'$ through a certain deformation process (cf. [2], § 3). In that case, we have the following:

Corollary 6.1. $\sigma(F_0)=\sum_{i=1}^r \sigma(F_i')$.

Remark. As we see above, each singular fiber behaves as if it has 'fractional signature'.

Proof of Theorem 6. Let $\omega: E\to X$ be any torus bundle over a connected surface X. Let $\partial X=C_1\cup\cdots\cup C_r$. Let $\alpha_i\in \mathrm{SL}(2,Z)$ be a monodromy matrix of the restriction $E\,|\,C_i$ of E to C_i, where C_i is oriented so that the orientation is concordant with that of X. The conjugacy class of α_i is uniquely determined.

Let ϕ: $\mathrm{SL}(2,Z)\to(1/3)Z$ be Meyer's class function whose explicit formula (containing the Dedekind sum) is found in [4], § 5.