11. *Fourier Coefficients of Siegel Cusp Forms of Degree Two*

By Yoshiyuki KITAOKA
Department of Mathematics, Nagoya University

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Our aim is to estimate Fourier coefficients of Siegel cusp forms of degree two under an assumption about the estimates of generalized Kloosterman sums.

Let H be the space of 2×2 complex symmetric matrices whose imaginary part is positive definite and $\Gamma = Sp_2(\boldsymbol{Z})$. $\Lambda = \{S \in M_2(\boldsymbol{Z}) \mid S = {}^tS\}$, $\Lambda^* = \{S \in M_2(\boldsymbol{Q}) \mid S = (s_{ij}),\ s_{ii} \in \boldsymbol{Z},\ 2s_{12} = 2s_{21} \in \boldsymbol{Z}\}$. $e(z)$ means $\exp(2\pi i z)$ for a complex number z.

Assumption. *Let $C \in M_2(\boldsymbol{Z})$, $|C| \neq 0$. For $G_1, G_2 \in \Lambda^*$ we put*

$$K(G_1, G_2; C) = \sum_D e(\operatorname{tr}(AC^{-1}G_1 + C^{-1}DG_2)),$$

where D runs over $\left\{D \in M_2(\boldsymbol{Z}) \bmod C\Lambda \,\middle|\, \begin{pmatrix} * & * \\ C & D \end{pmatrix} \in \Gamma \right\}$ and $A \in M_2(\boldsymbol{Z})$ is any matrix such that $\begin{pmatrix} A & * \\ C & D \end{pmatrix} \in \Gamma$. For these generalized Kloosterman sums we assume for $0 < \kappa \leq 1/2$,

$$K\left(G_1, G_2; \begin{pmatrix} c_1 & 0 \\ 0 & c_2 \end{pmatrix}\right) = O(c_1^2 c_2^{1-\kappa+\varepsilon}(c_2, g)^{\kappa}),$$

where $G_1, G_2 \in \Lambda^*$, $c_1 \mid c_2$ are natural numbers, ε is any positive number and g is the $(2,2)$-entry of G_2. ($\kappa = 1/2$ is plausible.)

Theorem. *Let k be an even integer ≥ 6. Let*

$$f(Z) = \sum_{0 < T \in \Lambda^*} a(T) e(\operatorname{tr} TZ)$$

be a cusp form of degree two, weight k. Suppose that Assumption is true, then we have

$$a(T) = O(|T|^{k/2-\kappa/2+\varepsilon}) \qquad \text{for any } \varepsilon > 0.$$

Sketch of the proof. Put $\Gamma_1 = \left\{\begin{pmatrix} 1_2 & S \\ 0 & 1_2 \end{pmatrix} \,\middle|\, S \in \Lambda \right\}$ and denote by $\mathfrak{h}$ the representatives of $\Gamma_1 \backslash \Gamma / \Gamma_1$ and put $\theta(M) = \left\{S \in \Lambda \,\middle|\, M\begin{pmatrix} 1_2 & S \\ 0 & 1_2 \end{pmatrix} M^{-1} \in \Gamma_1 \right\}$ for $M \in \Gamma$. By virtue of [1], [4] we may assume

$$f(Z) = g(Z, Q) = \sum_{M \in \Gamma_1 \backslash \Gamma} e(\operatorname{tr}(M\langle Z\rangle \cdot Q)) |CZ + D|^{-k},$$

where $0 < Q \in \Lambda^*$, $M = \begin{pmatrix} * & * \\ C & D \end{pmatrix}$. Then we have ([1])

$$g(Z, Q) = \sum_{M \in \mathfrak{h}} \sum_{0 < T \in \Lambda^*} h(M, T) e(\operatorname{tr} TZ),$$

where