37. *Potential Theory and Eigenvalues of the Laplacian*

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§1. **Introduction.** We consider a bounded domain Ω in $\mathbf{R}^3$ with C^2 boundary γ. We fix a point w in Ω. Let D be an open neighbourhood of the origin. Let $D(\varepsilon, w)$ be the set defined by $D(\varepsilon, w)=\{x \in \mathbf{R}^3;\ \varepsilon^{-1}(x-w) \in D\}$. We put $\Omega(\varepsilon)=\Omega\backslash\overline{D(\varepsilon, w)}$. Let $0>m_1(\varepsilon)\geq m_2(\varepsilon)\geq\cdots$ be the eigenvalues of the Laplacian in $\Omega(\varepsilon)$ under the Dirichlet condition on $\partial\Omega(\varepsilon)$. Let $0>m_1\geq m_2\geq\cdots$ be the eigenvalues of the Laplacian in Ω under the Dirichlet condition on γ. We arrange them repeatedly according to their multiplicities.

We proposed the following problem in Ozawa [1].

Problem. *Describe the precise asymptotic behaviour of $m_j(\varepsilon)$ as ε tends to zero.*

And the author conjectured in [1] the following

Conjecture. *Fix j. Assume that the multiplicity of m_j is one, then there exists a constant $c(D)$ such that*

$$m_j(\varepsilon)-m_j=-4\pi c(D)\varepsilon\varphi_j(w)^2+0(\varepsilon^{3/2}) \tag{1.1}$$

holds as ε tends to zero. Here $\varphi_j(x)$ is the normalized eigenfunction of the Laplacian associated with m_j.

In this note we give an answer to the above problem. We have the following

Theorem 1. *Under the same assumption as above,* (1.1) *holds and $c(D)$ is the electrostatic capacity* cap (D) *of the set D. Moreover,*

$$m_j(\varepsilon)-m_j+4\pi \operatorname{cap}(D)\varepsilon\varphi_j(w)^2=0(\varepsilon^{2-s}) \tag{1.2}$$

holds for an arbitrary fixed $s>0$ as ε tends to zero.

Remark. We define cap (D) by

$$\operatorname{cap}(D)=-(4\pi)^{-1}\int_{\partial D}\frac{\partial u}{\partial\nu}d\sigma, \tag{1.3}$$

where u is the unique solution of

$$\Delta u=0 \quad \text{in } D^c, \quad u|_{\partial D}=1, \quad \lim_{|x|\to\infty}u(x)=0. \tag{1.4}$$

Here $d\sigma$ is the surface element of ∂D.

The above Theorem 1 is a generalization of Theorem 2 in Ozawa [2]. The work in this paper was heavily inspired by the paper Papanicolau-Varadhan [5] in which "many holes problem" was studied.

In §2, we give an outline of the proof of Theorem 1. Details of this paper will be given elsewhere.