20. *Spatial Growth of Solutions of a Non-Linear Equation*

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1. Given a continuous function $M(u, \bar{u})$ of $(u, \bar{u}) \in [0, 1]^2$ and a nondecreasing function $F(x)$ on $\boldsymbol{R}=(-\infty, +\infty)$ with $\lim_{x\to-\infty} F(x)=0$, and $\lim_{x\to+\infty} F(x)=1$, let us consider the following evolution equation

$$(1) \qquad \frac{\partial u}{\partial t}=M(u, \bar{u}) \quad (u=u(x, t),\ x \in \boldsymbol{R},\ t>0)$$

where

$$\bar{u}=\bar{u}(x, t)=\int_{-\infty}^{+\infty} u(x-y, t)dF(y).$$

It is assumed throughout the paper that M has continuous partial derivatives $M_u=\partial M/\partial u$ and $M_{\bar{u}}=\partial M/\partial \bar{u}$, and satisfies

$$(2) \qquad \alpha \equiv M_{\bar{u}}(0, 0)>0, \qquad \beta \equiv M_u(0, 0)>-\alpha$$

$$(3) \qquad M(0, 0)=M(1, 1)=0\,;\ M_{\bar{u}}(u, \bar{u}) \geqq 0 \ \text{ for } (u, \bar{u}) \in [0, 1]^2$$

$$(4) \qquad M(u, u)>0 \qquad \text{for } 0<u<1,$$

and that F is right-continuous and satisfies

$$(5) \qquad 0<F(0-) \leqq F(0)<1$$

and its bilateral Laplace transform

$$\psi(\theta) \equiv \int_{-\infty}^{+\infty} e^{\theta x} dF(x)$$

is convergent in a neighborhood of zero.

It is routine to see from (3) that for any Borel measurable function $f(x)$ taking values in [0, 1], there is a unique solution of (1), with initial condition $u(x, 0)=f(x)$, which is also confined in [0, 1] (we will consider only such solutions), and that if two initial functions satisfy $0 \leqq f_1 \leqq f_2 \leqq 1$, the corresponding solutions preserve the inequality.

A typical example of M is $M(u, \bar{u})=\alpha\bar{u}-(\alpha+\beta)u\bar{u}+\beta u$. If we let $\beta=0$ in this example, (1) becomes the equation of simple epidemics (cf. [5])

$$(6) \qquad \frac{\partial u}{\partial t}=\alpha\bar{u}(1-u).$$

Another typical case is $M=\alpha(\bar{u}-u)+g(u)$, where g is continuously differentiable function with $g(0)=g(1)=0$, $g'(0)>0$ and $g(u)>0$ for $0<u<1$. If we replace, in this case, the compound Poisson operator $u \mapsto \bar{u}$ by the diffusion operator $u \mapsto \partial^2 u/\partial x^2$, a nonlinear diffusion equation