119. *Higher Order Nonsingular Immersions in Lens Spaces* Mod 3

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1. Introduction. H. Suzuki studied in [8] and [9] necessary conditions for the existence of higher order nonsingular immersions of projective spaces in projective spaces by making use of characteristic classes, γ-operations, spin operations, and mod 2 S-relations of stunted real projective spaces.

Let $L^n(q)$ be the $(2n+1)$-dimensional standard lens space mod q. A continuous map $f: L^n(q) \to L^m(q)$ is said to be of degree d $(\in Z_q)$ if $f^*x_m = dx_n$, where x_k is the distinguished generator of $H^2(L^k(q); Z_q)$ $(k=m, n)$ and $f^*: H^2(L^m(q); Z_q) \to H^2(L^n(q); Z_q)$ is the homomorphism induced by f. If $m>n$, there is a bijection of the set $[L^n(q), L^m(q)]$ of homotopy classes $[f]$ of continuous maps $f: L^n(q) \to L^m(q)$ onto the group Z_q defined by $[f] \to \deg f$ [5, Lemmas 2.6 and 2.7]. Hence, a continuous map $f: L^n(3) \to L^m(3)$ $(n<m)$ is homotopically non-trivial if and only if $\deg f = \pm 1$. The condition for the existence of homotopically trivial higher order nonsingular immersions of $L^n(q)$ is studied in [6] and [4]. In this paper we are concerned with homotopically non-trivial higher order nonsingular immersions of $L^n(3)$ in $L^m(3)$.

2. Notations and theorems. Let n and k be positive integers. Define an integer A as follows:

$$A = \sum_{j \in \Lambda} \binom{n+j}{j} \binom{n+k-j}{k-j},$$

where $\Lambda = \{j \in Z \mid 0 \leq j \leq (k-1)/2 \text{ and } 2j \not\equiv k \bmod 3\}$ and $\binom{m}{i} = m!/((m-i)!\,i!)$. For example, $A = n+1$ if $k=1$, $= \binom{n+2}{2}$ if $k=2$, $= (n+1)\binom{n+2}{2}$ if $k=3$, $= \binom{n+4}{4} + (n+1)\binom{n+3}{3}$ if $k=4$. Let $\nu = \nu(2n+1, k)$ denote the dimension $\binom{2n+1+k}{k} - 1$ of the fibre of the kth order tangent bundle $\tau_k(L^n(3))$ of $L^n(3)$.

Theorem 1. *Suppose there exists a homotopically non-trivial kth order nonsingular immersion of $L^n(3)$ in $L^m(3)$ with respect to dissections $\{D_i\}$ on $L^m(3)$.* (i) *If $2m+1 \geq \nu$, then* $\binom{m+1-A}{j} \equiv 0 \bmod 3$ *for m*