114. *Microlocal Analysis of Partial Differential Operators with Irregular Singularities*

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(Communicated by Kôsaku YOSIDA, M. J. A., Dec. 12, 1981)

We denote the variables in $M=\boldsymbol{R}^{n+1}$ by $x=(x_0, x')$, where $x_0 \in \boldsymbol{R}$ and $x' \in \boldsymbol{R}^n$. We investigate partial differential operators of the form

$$P(x, \partial/\partial x)=\sum_{|\alpha|\leqslant m} a_\alpha(x) x_0^{\kappa(|\alpha|)}(\partial/\partial x)^\alpha$$

microlocally at $\mathring{x}^*=(0; \sqrt{-1}, 0, \cdots, 0) \in \sqrt{-1}T^*\boldsymbol{R}^{n+1}$. Here $a_\alpha(x)$, $|\alpha| \leqslant m$, are real analytic in a neighborhood of $x=0$, $a_{(m,0,\cdots,0)}=1$, and $\kappa(j)$, $0 \leqslant j \leqslant m$, are some integers $\geqslant 0$.

Definition 1. After Aoki [3], we define the *irregularity* σ of $P(x, \partial/\partial x)$ by

$$\sigma=\max\left\{\max_{0\leqslant j\leqslant m-1}\left(\frac{\kappa(m)-\kappa(j)}{m-j}\right), 1\right\}.$$

If $\sigma=1$, Kashiwara and Oshima [5] called the above operator $P(x, \partial/\partial x)$ a partial differential operator with regular singularities along the hypersurface $N=\{x_0=0\}$. They proved, in this case, that the above operator $P(x, \partial/\partial x)$ is equivalent to the very simple operator

$$\begin{array}{ccc} x_0^{\kappa(m)}: \mathcal{C}_M & \longrightarrow & \mathcal{C}_M, \\ \cup\!\!\!\text{|} & & \cup\!\!\!\text{|} \\ u & \longmapsto & x_0^{\kappa(m)}u \end{array}$$

microlocally at $\mathring{x}^*$.

Our purpose is to generalize this result to the case $\sigma>1$. If $\sigma>1$, we say that the above operator has irregular singularities along the hypersurface N.

Definition 2. Let $\sigma>1$. We denote by $\lambda_1, \cdots, \lambda_{\kappa(m)}$ the roots of the algebraic equation

$$\lambda^{\kappa(m)}+\sum_{\pi(P)} a_{(j,0,\cdots,0)}(0)\lambda^{\kappa(j)}=0,$$

where

$$\pi(P)=\left\{0\leqslant j\leqslant m-1; \frac{\kappa(m)-\kappa(j)}{m-j}=\sigma\right\}.$$

We call these constants the *characteristic exponents* of P.

We investigate such a type of operators by means of holomorphic microlocal operators, due to Sato, Kawai and Kashiwara [7] and Aoki [2]. Now we have the following

Theorem 1. *Assume that* $\sigma>1$ *and that*

$$\lambda_i \neq \lambda_j \qquad \text{if } i\neq j.$$