100. *An Average Type Result on the Number of Primes Satisfying Generalized Wieferich Condition*

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1. Statement of results. In 1909 Wieferich ([1]) proved that if an odd prime p satisfies the condition

$$2^{p-1}-1 \not\equiv 0 \pmod{p^2},$$

then the case I of Fermat's Last Theorem is true for this prime p, i.e. under the condition $(xyz, p)=1$, there exists no integral solution for the Diophantine equation $x^p+y^p=z^p$. Moreover, it is now known (see for example [2]) that we can deduce the same conclusion, if an odd prime p satisfies

$$a^{p-1}-1 \not\equiv 0 \pmod{p^2}$$

for some prime value a, $2 \leqslant a \leqslant 43$.

Now we shall call

$$(*) \qquad a^{p-1}-1 \equiv 0 \pmod{p^2}$$

the generalized Wieferich condition for a (a may be any natural number). We define for real $x>0$,

$$F_a(x)=\{p\,;\, p \text{ is an odd prime} \leqslant x, p \text{ satisfies } (*)\}.$$

We have an average type result as to the cardinal $\sharp F_a(x)$ of $F_a(x)$, which states as follows:

Theorem 1. *Let δ be an arbitrary fixed real number satisfying $1/2<\delta<1$. We have, if $x \geqslant 286$,*

$$\sharp F_a(x)=\log\log x+\theta((\log\log x)^{\delta})+\left(C-\frac{1}{2}\right)+\frac{1}{2}\theta((\log x)^{-2})$$

for all a such that $2 \leqslant a \leqslant x^4$ with at most

$$2x^4(\log\log x)^{1-2\delta}$$

exceptions of a, where $C=\gamma+\sum_{p:\text{ prime}}\{\log(1-1/p)+1/p\}$ and γ is Euler's constant. ($f(x)$ being positive valued function of x, $\theta(f(x))$ denotes a function of x whose absolute value $\leqslant f(x)$.)

Similarly we have:

Theorem 2. *Let D be an arbitrary fixed real number >0 and $y \geqslant x^6$. We defined for a natural number a and real $x>0$,*

$$F_a^{(3)}(x)=\{p\,;\, p \text{ is an odd prime} \leqslant x,\ a^{p-1}-1\equiv 0 \pmod{p^3}\}.$$

Then we have

$$\left|\sharp F_a^{(3)}(x)-\sum_{\substack{3\leqslant p\leqslant x\\ p:\text{ prime}}}\frac{1}{p^2}\right|<D$$