# 98. *On Unramified $SL_2(F_p)$ Extensions of an Algebraic Function Field of Genus 2*

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Let $k$ be an algebraically closed field of characteristic $p$. Let $K$ be an algebraic function field over $k$. In [1], we calculated the number of unramified $SL_2(F_4)$ extensions of some algebraic function field of characteristic 2. In this note, we obtain the best possible estimation of the number of unramified Galois extensions of $K$ whose Galois groups are isomorphic to $SL_2(F_p)$ when $p \geqq 5$ and the genus of $K$ is 2. Detailed accounts will be published elsewhere.

Definition 1. For any system $(m_1, \cdots, m_r; n_1, \cdots, n_s)$ of integers such that $m_1 = n_1$, we define the number $N(m_1, \cdots, m_r; n_1, \cdots, n_s)$ inductively as follows:

(1) When $r = s$, we define

$$N(m_1, \cdots, m_r; n_1, \cdots, n_r) = \sum_{i=1}^{r} m_i + \sum_{i=1}^{r} n_i - r m_1.$$

(2) When $r < s$, first we define

$$N(m_1, n_1, \cdots, n_s) = n_1 \cdots n_s.$$

Assume that for $(m_1, \cdots, m_r; n_1, \cdots, n_{s-1})$ and $(m_1, \cdots, m_{r-1}; n_1, \cdots, n_{s-1})$ the number $N(\cdots)$ is defined. Then we define

$$N(m_1, \cdots, m_r; n_1, \cdots, n_s) = (m_r + n_s - m_1) N(m_1, \cdots, m_r; n_1, \cdots, n_{s-1}) + N(m_1, \cdots, m_{r-1}; n_1, \cdots, n_{s-1}).$$

(3) When $r > s$, we define

$$N(m_1, \cdots, m_r; n_1, \cdots, n_s) = N(n_1, \cdots, n_s; m_1, \cdots, m_r).$$

Let $K$ be the maximum unramified Galois extension of $K$. We put $q = p^m$, where $m$ is a natural number. We denote by Irr (Gal $(\tilde{K}/K)$, $SL_2(F_q)$) the set of $GL_2(k)$ equivalence classes of 2-dimensional irreducible representations of Gal $(\tilde{K}/K)$ whose images are isomorphic to subgroups of $SL_2(F_q)$. Let $\rho$ be a representative of an element of Irr (Gal $(\tilde{K}/K)$, $SL_2(F_p)$). Then we note that the order $\sharp\rho$ (Gal $(\tilde{K}/K)$) is prime to $p$ if $\rho$ (Gal $(\tilde{K}/K)) \subsetneqq SL_2(F_p)$. Hence to estimate the number of unramified $SL_2(F_p)$ extensions of $K$, it suffices to estimate

$$\sharp \mathrm{Irr}\,(\mathrm{Gal}\,(\tilde{K}/K), SL_2(F_p)).$$

Theorem 1. *We put*

$$A = N(p+\overbrace{1, \cdots, p}^{p+2}+1, \overbrace{p, \cdots, p}^{p-3}; p+\overbrace{1, \cdots, p}^{p}+1, \overbrace{p, \cdots, p}^{p+1})$$