93. *The Stability Theorems for Discrete Dynamical Systems on Two-Dimensional Manifolds*

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(Communicated by Kôsaku YOSIDA, M. J. A., Oct. 12, 1981)

0. Introduction. One of the basic problems in the theory of dynamical systems is the characterization of stable systems. Let M be a closed (i.e. compact connected without boundary) smooth manifold with a smooth Riemannian metric and $\mathrm{Diff}^r(M)$ denote the space of C^r diffeomorphisms on M with the uniform C^r topology for $r\geq 1$. Let $f\in \mathrm{Diff}^s(M)$ for $s\geq r$. Then f is called C^r *structurally stable* if and only if there is a neighborhood $\mathcal{U}(f)$ of f in $\mathrm{Diff}^r(M)$ such that for any $g\in\mathcal{U}(f)$ there exists a homeomorphism $h: M\to M$ satisfying $gh=hf$. Another important notion of stability is Ω-stability. We denote by $\Omega(f)$ the non-wandering set of f. f is called C^r *Ω-stable* if and only if there is a neighborhood $\mathcal{U}(f)$ of f in $\mathrm{Diff}^r(M)$ such that for any $g\in\mathcal{U}(f)$ there exists a homeomorphism $h: \Omega(f)\to\Omega(g)$ satisfying $gh=hf$ on $\Omega(f)$.

The essential condition to characterize these stabilities is "Axiom A", namely f satisfies *Axiom A* if and only if

(a) $\Omega(f)$ is a hyperbolic set,

(b) $\overline{\mathrm{Per}(f)}=\Omega(f)$,

where $\mathrm{Per}(f)$ denotes the set of all periodic points of f. Recall that a compact f-invariant subset $\Lambda\subset M$ is a *hyperbolic set* if and only if there exist constants $c>0$, $0<\lambda<1$ and a Tf-invariant continuous splitting $TM|\Lambda=E^s\oplus E^u$ such that

$$\|Tf^n|E^s_p\|\leq c\lambda^n,\qquad \|Tf^{-n}|E^u_p\|\leq c\lambda^n,$$

for all $p\in\Lambda$ and non-negative integer n.

In [5], [11] and [4], the following are conjectured.

Structural stability conjecture. *f is C^r structurally stable if and only if f satisfies Axiom A and Strong transversality condition.*

Ω-stability conjecture. *f is C^r Ω-stable if and only if f satisfies Axiom A and No-cycle property.*

For the definitions of Strong transversality condition and No-cycle property, we refer to [5] and [11].

The pupose of this paper is to give an affirmative answer to these conjectures for f of class C^2 in case of $\dim M=2$ and $r=1$.

The sufficiency of Axiom A and Strong transversality condition (resp. No-cycle property) for structural (resp. Ω-) stability has been