92. *Characteristic Boundary Value Problems for Hyperbolic Equations*

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§ 1. **Problems and results.** We study a priori estimates of the solution u of boundary value problems in the half space:

$$(1.1)\qquad \begin{cases} P(D_x, D_z)u(x,z)=f(x,z) & \text{for } x>0,\ z\in \boldsymbol{R}^n, \\ B_j(D_x, D_z)u|_{x=0}=g_j(z) & \text{for } z\in \boldsymbol{R}^n,\ j=0,1,\cdots,\mu^+-1. \end{cases}$$

Let $(x,z)=(x,y,t)$ denote the variables in $\boldsymbol{R}_x\times\boldsymbol{R}_y^{n-1}\times\boldsymbol{R}_t$ and $(\xi,\zeta)=(\xi,\eta,\tau)$ denote the covariables corresponding to $(D_x, D_y, D_t)=(\partial/i\partial x, \partial/i\partial y, \partial/i\partial t)$. We assume

(P1) $P(D)=P(D_x, D_y, D_t)$ is a homogeneous differential operator of order m with constant coefficients and strictly hyperbolic with respect to D_t, and

(P2) the boundary $\{x=0\}$ is characteristic to P, i.e.

$$(1.2)\qquad P^0(1,0,0)=0,$$

where $P^0(\xi,\eta,\tau)$ is the principal symbol of $P(\xi,\eta,\tau)$.

There exists from (P1) a positive constant γ_0 such that

$$(1.3)\qquad P(\xi,\eta,\tau)\neq 0 \qquad \text{for } (\xi,\eta,\tau)\in \boldsymbol{R}^n\times\{\operatorname{Im}\tau<-\gamma_0\}.$$

Moreover, thanks to (P1) and (P2), we have an expression

$$(1.4)\qquad P(D_x, D_y, D_t)=P_{m-1}(D_z)D_x^{m-1}+P_{m-2}(D_z)D_x^{m-2}+\cdots+P_0(D_z),$$

where

$$(1.5)\qquad P_{m-1}(\eta,\tau)\neq 0 \qquad \text{for } (\eta,\tau)\in \boldsymbol{R}^{n-1}\times\{\operatorname{Im}\tau<-\gamma_0\}.$$

From (1.3) and (1.5), we have always $m-1$ non real roots of the characteristic equation $P(\xi,\zeta)=0$ for parameters $\zeta\in\boldsymbol{R}^{n-1}\times\{\operatorname{Im}\tau<-\gamma_0\}$. We denote them by $\xi_1^+(\zeta),\cdots,\xi_{\mu^+}^+(\zeta), \xi_1^-(\zeta),\cdots,\xi_{\mu^-}^-(\zeta)$, where $\pm$ mean the sign of imaginary parts. $\mu^+ + \mu^- = m-1$, and $\mu^\pm$ are independent on the parameter ζ.

We introduce boundary operators

$$(1.6)\qquad B_j(D_x, D_z)=\textstyle\sum_{k=0}^{m_j} B_{j,k}(D_z)D_x^k$$

of total order b_j $(m_j\leqq b_j)$. We assume

(B1) the number of boundary conditions is μ^+ i.e. $j=0,1,\cdots,\mu^+-1$ in (1.6), and

(B2) $0\leqq m_j\leqq m-1$ i.e. $m_*=\max_j m_j<m-1$.

We define a polynomial of ξ with paramter ζ by

$$(1.7)\qquad \begin{aligned} P^+(\xi,\zeta)&=\textstyle\sum_{j=1}^{\mu^+}(\xi-\xi_j^+(\zeta)) \\ &=P_\mu^+(\zeta)\xi^{\mu^+}+\cdots+P_1^+(\zeta)\xi+P_0^+(\zeta). \end{aligned}$$

Dividing $B_j(\xi,\zeta)$ by $P^+(\xi,\zeta)$ as polynomials of ξ, we have the quotient