89. A Variational Problem Relating to the Theory of Optimal Economic Growth*)

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1. Introduction. The theory of optimal economic growth is one of the most attractive themes in the recent developments in mathematical economics. The basic problem is to find out an optimal path of economic growth (or capital accumulation) in the sense that it maximizes certain economic welfare over time under some technological constraint. Being stimulated by the ingenious idea of F. P. Ramsey [6], a lot of economists, including P. A. Samuelson and T. C. Koopmans, have been working on this field and various mathematical theories of optimal control such as the Pontrjagin's maximum principle have been successfully introduced to economic analysis.

Recently, Chichilnisky [2] tried to prove rigorously the existence of an optimal path of economic growth relying upon an effective use of the weighted Sobolev space. And Takekuma [7] also gave another interesting version of the existence proof. The purpose of the present paper is to add a further new insight to this existence problem, and the author is much indebted to Berkovitz [1] for the basic ideas embodied in the proof.

2. Problem. Let us begin with specifying some notations and their economic interpretations. First the following items are assumed to be given.

$[0, T]$ planning time horizon.
$u: \boldsymbol{R}_+^l \to \boldsymbol{R}_+$ welfare function.
$f: \boldsymbol{R}_+^l \to \boldsymbol{R}_+^l$ production function at time 0.
$\rho > 0$ the rate of technological progress.
$\delta > 0$ the discount rate of the welfare in the future.
$\lambda \in (0, 1)^l$ the vector of the depreciation rates of l capital goods.

Furthermore we have a couple of variable mappings to be optimized:

$k: [0, T] \to \boldsymbol{R}_+^l$ path of capital accumulation.
$s: [0, T] \to [0, 1]^l$ path of the vector whose components are saving rates of each goods.

For any vector $x \in \boldsymbol{R}^l$, we designate by M_x the diagonal matrix of the form

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