77. *On Ranked Linear Spaces. II*

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We shall explain in this note in detail the completion of the ranked linear spaces defined in I, as mentioned in I, § 1. The references here are the same as those in I[*)].

§ 5. Completion of ranked linear spaces. Definition. Let $(E, \mathfrak{V}_{E,n}, \mathfrak{V}_{E,n}^{(p)})$ be a separated ranked linear space and $(\hat{E}, \hat{\mathfrak{V}}_{\hat{E},n}, \hat{\mathfrak{V}}_{\hat{E},n}^{(\hat{p})})$ a complete ranked linear space. $(\hat{E}, \hat{\mathfrak{V}}_{\hat{E},n}, \hat{\mathfrak{V}}_{\hat{E},n}^{(\hat{p})})$ is said to be a *completion* of $(E, \mathfrak{V}_{E,n}, \mathfrak{V}_{E,n}^{(p)})$ if $(E, \mathfrak{V}_{E,n}, \mathfrak{V}_{E,n}^{(p)})$ is equivalent to a ranked linear subspace of $(\hat{E}, \hat{\mathfrak{V}}_{\hat{E},n}, \hat{\mathfrak{V}}_{\hat{E},n}^{(\hat{p})})$ which is dense in $(\hat{E}, \hat{\mathfrak{V}}_{\hat{E},n}, \hat{\mathfrak{V}}_{\hat{E},n}^{(\hat{p})})$.

We shall now construct a completion of a given separated ranked linear space $(E, \mathfrak{V}_{E,n}, \mathfrak{V}_{E,n}^{(p)})$.

Let us denote by M' the family of canonical fundamental sequences in $(E, \mathfrak{V}_{E,n}, \mathfrak{V}_{E,n}^{(p)})$. We introduce in M' an equivalence relation ρ defined as follows:

For $u, v \in M'$, $u\rho v$ iff there exists a $w \in M'$ satisfying $u \prec w$ and $v \prec w$. (This is the same equivalence relation as that used in [7] and [9].) Remark that for two $u=\{p_i+U_i\}$ and $v=\{q_i+V_i\}$ in M' it holds that $u\rho v$ iff $\{p_i-q_i\}\to 0$.

Then we have

Lemma 1. *If* $u\cap v\neq\phi$ *for* $u, v\in M'$, *then* $u\rho v$. (Here $u\cap v\neq\phi$ means $(p_i+U_i)\cap(q_i+V_i)\neq\phi$ for all i where $u=\{p_i+U_i\}$ and $v=\{q_i+V_i\}$.)

Furthermore,

Lemma 2. *For any* $u=\{p_i+U_i\}$, $v=\{q_i+V_i\}$ *in* M' *and any scalar* $\alpha\neq 0$, *there exist* w *and* w' *in* M' *such that* $u+v\prec w$ *and* $\alpha u\prec w'$. (Here $u+v$ and αu mean the sequences $\{p_i+U_i+q_i+V_i\}$ and $\{\alpha p_i+\alpha U_i\}$ respectively.)

By virtue of Lemma 2, we can define linear operations in $\tilde{M}=M'/\rho$. (Hereafter we shall denote by $\tilde{u}$ the equivalence class that include $u\in M'$.) For $\tilde{u}, \tilde{v}\in\tilde{M}$ and a scalar $\alpha\neq 0$, there exist w and w' in M' such that $u+v\prec w$ and $\alpha u\prec w'$ by Lemma 2, we define $\tilde{w}=\tilde{u}+\tilde{v}$ and $\tilde{w}'=\alpha\tilde{u}$.

Let τ be a mapping of E into $\tilde{M}$ such that $\tau(p)=\tilde{u}_p$, where u_p is a p-canonical fundamental sequence.

Lemma 3. *The mapping* $\tau: E\to\tau(E)$ *is linear and one-to-one.*

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*) Teruko Tsuda, On ranked linear spaces I. Proc. Japan Acad., 57A, 262–266 (1981).