75. On a Certain Decomposition of 2-Dimensional Cycles on a Product of Two Algebraic Surfaces

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In this note, we define a type of decomposition for the 4-dimensional cohomology group of a product of two algebraic surfaces and we use such a decomposition for investigation of algebraic 2-cycles on it. Details of this note will appear elsewhere.

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§ 1. **Hodge-Künneth-Transcendence-decomposition.** Let S and S' be non-singular projective surfaces defined over the field of complex numbers $\boldsymbol{C}$. We denote by $C^r(S\times S')$ the group of all cycles of codimension r on $S\times S'$ modulo rational equivalence, and we have a cycle map cl, which to each cycle $X\in C^r(S\times S')\otimes_{\boldsymbol{Z}}\boldsymbol{Q}$ associates the cohomology class $cl(X)\in H^{2r}(S\times S',\boldsymbol{C})$. Let $H^{2r}(S\times S',\boldsymbol{Q})_{\rm alg}$ denote the image of $cl: C^r(S\times S')\otimes_{\boldsymbol{Z}}\boldsymbol{Q}\to H^{2r}(S\times S',\boldsymbol{C})$. Then, using the Hodge decomposition

(1.1) $$H^{2r}(S\times S',\boldsymbol{C})\cong\bigoplus_{p+q=2r}H^{p,q}(S\times S',\boldsymbol{C})$$

of the complex cohomology, we know

$$H^{2r}(S\times S',\boldsymbol{Q})_{\rm alg}\subseteqq H^{r,r}(S\times S',\boldsymbol{C})\cap H^{2r}(S\times S',\boldsymbol{Q})=H^{2r}(S\times S',\boldsymbol{Q})_{\rm Hodge}.$$

We define

$$H^2(S,\boldsymbol{C})_{\rm trans}=\varinjlim_{U\subset S,\,{\rm open}}H^2(U,\boldsymbol{C}),$$

and we have the "transcendence-decomposition" of $H^2(S,\boldsymbol{C})$ with respect to the intersection numbers,

(1.2) $$H^2(S,\boldsymbol{C})\cong H^2(S,\boldsymbol{C})_{\rm alg}\oplus H^2(S,\boldsymbol{C})_{\rm trans}$$

where $H^2(S,\boldsymbol{C})_{\rm alg}=H^2(S,\boldsymbol{Q})_{\rm alg}\otimes_{\boldsymbol{Q}}\boldsymbol{C}$ (cf. Hodge and Atiyah [3], Grothendieck [1]).

Using (1.1), (1.2) and the Künneth decomposition, we make the following

Definition (1.3). The Hodge-Künneth-Transcendence-part (HKT-part) of $H^4(S\times S',\boldsymbol{C})$ is its subspace

$$H^4_{\rm hkt}(S,S')\cong\{H^{2,0}(S,\boldsymbol{C})\otimes H^{0,2}(S',\boldsymbol{C})\}\oplus\{H^{0,2}(S,\boldsymbol{C})\otimes H^{2,0}(S',\boldsymbol{C})\}$$
$$\oplus\{H^{1,1}(S,\boldsymbol{C})_{\rm trans}\otimes H^{1,1}(S',\boldsymbol{C})_{\rm trans}\},$$

where $H^{1,1}(S,\boldsymbol{C})_{\rm trans}=H^{1,1}(S,\boldsymbol{C})\cap H^2(S,\boldsymbol{C})_{\rm trans}$. We let $p: H^4(S\times S',\boldsymbol{C})\to H^4_{\rm hkt}(S,S')$ denote the projection, and let $H^4_{\rm hkt}(S,S')_{\rm alg}=H^4_{\rm hkt}(S,S')\cap H^4(S\times S',\boldsymbol{Q})_{\rm alg}$.