73. On Cayley-Aronhold Realizations of sl$(n+1, K)$

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1. In the present note, we shall sketch an outline of invariant theory of formal power series in $(r+1)\times(n-r)$-variable matrix $z=(z_{\alpha i})_{0\leq\alpha\leq r, r+1\leq i\leq n}$ with respect to sl $(n+1, K)$.

First, we shall list notations freely used:

K: a fixed field of characteristic zero,

n: a positive integer,

r: a non-negative integer satisfying $0\leq r\lneqq n$,

$\alpha, \beta, \gamma, \cdots, \alpha', \beta', \gamma', \cdots$ run over $\{0, 1, 2, \cdots, r\}$,

$a, b, c, \cdots, a', b', c', \cdots$ run over $\{r+1, r+2, \cdots, n\}$,

$z=\begin{pmatrix} z_{0r+1}, & \cdots, & z_{0n} \\ \vdots & & \vdots \\ z_{rr+1}, & \cdots, & z_{rn} \end{pmatrix}$: a variable $(r+1)\times(n-r)$-matrix,

$\varepsilon_{\alpha a}$: the specialization of z such that

$$z_{\beta b}\longmapsto\begin{cases}1 & (\beta, b)=(\alpha, a),\\ 0 & (\beta, b)\neq(\alpha, a),\end{cases}$$

$\mathcal{L}=\left\{l=\begin{pmatrix} l_{0r+1}, & \cdots, & l_{0n} \\ \vdots & & \vdots \\ l_{rr+1}, & \cdots, & l_{rn} \end{pmatrix}\middle| l_{\alpha a} \text{ run over non-negative integers}\right\}$,

$z^l=\prod_{\alpha,a} z_{\alpha a}^{l_{\alpha a}}$,

$\left(\frac{\partial}{\partial z}\right)^l=\prod_{\alpha,a}\left(\frac{\partial}{\partial z_{\alpha a}}\right)^{l_{\alpha a}}$,

$l!=\prod_{\alpha,a} l_{\alpha a}!,\qquad \sum l=\sum_{\alpha,a} l_{\alpha a}$,

$e_{\alpha\beta}, e_{\alpha a}, e_{a\alpha}, e_{ab}$: the $(n+1)\times(n+1)$-matrices whose only non-zero entries are, respectively, the $(\alpha, \beta), (\alpha, a), (a, \alpha), (a, b)$-entries with value one.

Remark.

$$(l+\varepsilon_{\alpha a})!=l!\,(l_{\alpha a}+1)$$

$$(l+\varepsilon_{\alpha a}+\varepsilon_{\beta b})!=\begin{cases} l!\,(l_{\alpha a}+1)(l_{\alpha a}+2) & (\alpha, a)=(\beta, b),\\ l!\,(l_{\alpha a}+1)(l_{\beta b}+2) & (\alpha, a)\neq(\beta, b),\end{cases}$$

$$\sum(l+\varepsilon_{\alpha a})=\sum l+1,\quad \sum(l+\varepsilon_{\alpha a}-\varepsilon_{\beta b})=\sum l\quad (l_{\beta b}\geq 1).$$

2. We denote by $\xi=(\xi^{(l)})_{l\in\mathcal{L}}$ a vector of infinite length with indeterminate entries, and choose an element $w\neq 0, 1, 2, \cdots$ in K. The basic formal power series is defined by