69. *On the Solvability of Goursat Problems and a Function of Number Theory*

By Masafumi YOSHINO
Department of Mathematics, Tokyo Metropolitan University

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1. **Introduction.** In this paper we shall study the reduced Goursat problem with constant coefficients:

(1.1) $$Lu=(a\partial_1^{-1}\partial_2+\varepsilon+b\partial_1\partial_2^{-1}+c\partial_1^2\partial_2^{-2})u=h(x)$$

where $x=(x_1, x_2)\in C^2$, $\partial_i=\partial/\partial x_i$ $(i=1,2)$ and ∂_i^{-1} is the integration with respect to the variable x_i from the origin to x_i.

If the roots $\lambda_1, \lambda_2, \lambda_3$ of the characteristic equation of L;

(1.2) $$a\lambda^3+\varepsilon\lambda^2+b\lambda+c=0$$

satisfy the "Alinhac-Leray condition" $|\lambda_1|\leqq|\lambda_2|<|\lambda_3|$ the solvability and the uniqueness of (1.1) are proved by S. Alinhac in [1] under some additional conditions. Whereas, if the condition is not satisfied few results are known. The best work known is that of Leray's for (1.1) with $c=0$ in [2]. He introduced the number-theoretical function $\rho(\theta)$ (cf. [2]) and expressed a sufficient condition for the solvability and uniqueness of (1.1) for $c=0$ in terms of $\rho(\theta)$.

The purpose of this paper is to study the case $c\neq 0$ without assuming the Alinhac-Leray condition. We introduce a function $\rho(\theta_1, \theta_2)$ as a natural extension of the Leray's auxiliary function $\rho(\theta)$ which describes the transcendency of θ_1 and θ_2. In terms of this function we shall characterize the range of the operator L. As a result we reveal a close connection between the algebraic-transcendental properties of the characteristic roots and the solvability and uniqueness. We remark that the results here can be extended to a wider class of equations with multiple characteristic roots.

2. **Statement of theorems.** Without loss of generality we may assume that $ac\neq 0$. Moreover, by the linear change of variables such as $rx_1=z_1$, $x_2=z_2$ $(r\neq 0)$ we may assume that eq. (1.2) has the root 1 and that the absolute values of other roots do not exceed 1. Since we are interested in the case where the Alinhac-Leray condition is not satisfied we assume $0<|\lambda_1|\leqq|\lambda_2|=1$. Let H_0 be the set of functions analytic at the origin. Then

Theorem 2.1. *If the roots $\lambda_1, \lambda_2, 1$ of eq.* (1.2) *are not distinct the map $L: H_0\to H_0$ is bijective.*

In view of this theorem we shall consider the case where the roots $\lambda_1, \lambda_2, 1$ are distinct. Let I_k be defined by