41. *A Characterization of Homogeneous Self-Dual Cones*

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§ 1. It is known that every homogeneous self-dual cone is a Riemannian symmetric space of non-positive sectional curvature with respect to the canonical Riemannian metric (cf. [2], [4]). In 1965, Prof. Y. Matsushima raised the question whether every Riemannian symmetric homogeneous convex cone is self-dual or not. The purpose of the present note is to announce an affirmative answer to the above question. Furthermore, we will give an application. The detailed results with their complete proofs will be published elsewhere [6].

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§ 2. In the present note, we will employ the following notations and terminologies. Let V be a homogeneous convex cone in the n-dimensional real number space $\boldsymbol{R}^n$ with an inner product $\langle\ ,\ \rangle$. We denote by V^* the dual cone of V with respect to this inner product $\langle\ ,\ \rangle$. A cone V is called *self-dual* if the dual cone V^* with respect to a suitable inner product coincides with V. The *characteristic function* φ_V of V is defined by

$$\varphi_V(x)=\int_{V^*}\exp(-\langle x, y\rangle)dy$$

for every $x\in V$, where dy is a canonical Euclidean measure on $\boldsymbol{R}^n$. Let us take a system of linear coordinates $(x_1, x_2, \cdots, x_n)$ of $\boldsymbol{R}^n$. Then we can define a $G(V)$-invariant Riemannian metric g on V by

$$g=\sum_{i,j}\frac{\partial^2\log\varphi_V}{\partial x_i\partial x_j}dx_i dx_j,$$

where $G(V)=\{A\in GL(n, \boldsymbol{R}); AV=V\}$. This Riemannian metric g is called the *canonical metric* of V.

§ 3. It is known in [7] that there exists a natural bijection between the set of all linear equivalence classes of homogeneous convex cones and the set of all isomorphism classes of T-algebras. We recall briefly this bijection. (For the details, see [7].)

Let $\mathfrak{A}=\sum_{1\leq i,j\leq r}\mathfrak{A}_{ij}$ be a T-algebra of rank r with an involution $*$. We put

$$T(\mathfrak{A})=\{t=(t_{ij})\in\mathfrak{A};\ t_{ii}>0\ \text{and}\ t_{ij}=0\ \text{for}\ i>j\}$$

and

$$V(\mathfrak{A})=\{tt^*;\ t\in T(\mathfrak{A})\}.$$