35. *Normal Forms of Quasihomogeneous Functions with Inner Modality Equal to Five*

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§ 1. **Introduction.** In [3], K. Saito introduced the invariant $s(f)$ into quasihomogeneous functions with an isolated critical point 0, which is defined as the maximal quasi-degree of generators of a monomial base of the local ring $\mathcal{O}_{C^n}/(\partial f/\partial x_1, \cdots, \partial f/\partial x_n)$ (this local ring is denoted by R_f). And he classified quasihomogeneous functions with $s(f)=0$ and 1.

In [1], V. I. Arnol'd introduced the invariant $m_0(f)$ into quasihomogeneous functions with an isolated critical point 0, which is called the inner modality and defined as the number of generators of a monomial base of R_f on the Newton diagram and above it. And he classified quasihomogeneous functions with $m_0(f)=0$ and 1.

In [4], we classified quasihomogeneous functions with inner modality equal to 2, 3 and 4 and studied the relations of some adjaciencies among them.

In this paper, we shall give the classification of quasihomogeneous functions with inner modality equal to 5 and some adjaciencies among them.

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§ 2. **Classification.** Let a formal power series $f \in C[[x_1, \cdots, x_n]]$ be quasihomogeneous of type $(1; r_1, \cdots, r_n)$, i.e. the quasidegree of each monomial of f is equal to 1. This definition is equivalent to

$$f(t^{r_1}x_1, \cdots, t^{r_n}x_n)=tf(x_1, \cdots, x_n) \qquad \text{for any } t \in C.$$

K. Saito showed in [2] that if f has an isolated critical point 0, then there exists a coordinate system $(y_1, \cdots, y_n)$ such that $f=h(y_1, \cdots, y_k)+y_{k+1}^2+\cdots+y_n^2$, where h is a quasihomogeneous polynomial of type $(1; s_1, \cdots, s_k)$ $(0<s_j<1/2,\ j=1, \cdots, k,\ s_j \in Q)$. Then we call the natural number k the corank of f and call the polynomial h the residual part of f. In what follows, we may consider a quasihomogeneous polynomial of type $(1; r_1, \cdots, r_n)$ $(0<r_j\leqq 1/2,\ r_j \in Q)$ with an isolated critical point 0.

Definition 1 (Arnol'd [1]). Let f be as above. The inner modality of f is defined by the number of generators of a monomial base of R_f with quasi-degree equal to 1 and it is denoted by $m_0(f)$.