

23. On the Equivalence of Certain Comparison Theorems for Holonomic Systems of Differential Equations

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Introduction. In relation to the regular singularities in the theory of linear partial differential equations, J.-P. Ramis [6] and Z. Mebkhout [4] proposed three equivalent conditions of "GAGA" type for holonomic systems of linear differential equations. The purpose of this note is to show how these conditions are related to each other in the framework of duality between holonomic systems of differential equations and their solution sheaves as is developed in a recent work of M. Kashiwara and T. Kawai [2].

Let X be a complex manifold with structure sheaf \mathcal{O}_X . We denote by \mathcal{D}_X (resp. by \mathcal{D}_X^∞) the Ring of linear partial differential operators of finite order (resp. of infinite order) with holomorphic coefficients.

Let M be a bounded complex of (left) \mathcal{D}_X -Modules with holonomic cohomology and let Y be a closed analytic subset of X . We will give a simple proof of the equivalence of the following three conditions:

(i) The natural morphism

$$\alpha: \mathbf{R}\underline{\mathrm{Hom}}_{\mathcal{D}_X}(M, \mathcal{O}_X)_{X|Y} \longrightarrow \mathbf{R}\underline{\mathrm{Hom}}_{\mathcal{D}_X}(M, \mathcal{O}_{\hat{X}|Y})$$

is an isomorphism, where $\cdot_{X|Y}$ is the functor i_*i^{-1} , i being the inclusion mapping $Y \hookrightarrow X$, and $\cdot_{\hat{X}|Y}$ is the formal completion along Y .

(ii) The natural morphism

$$\beta: \mathbf{R}\underline{\mathrm{Hom}}_{\mathcal{D}_X}(\mathcal{O}_X, \mathbf{R}\underline{\Gamma}_{[Y]}(M)) \longrightarrow \mathbf{R}\underline{\Gamma}_Y(\mathbf{R}\underline{\mathrm{Hom}}_{\mathcal{D}_X}(\mathcal{O}_X, M))$$

is an isomorphism, where $\mathbf{R}\underline{\Gamma}_{[Y]}(M)$ is the algebraic local cohomology of M with supports in Y . (See Kashiwara [1].)

(iii) The natural morphism

$$\gamma: \mathcal{D}_X^\infty \otimes_{\mathcal{D}_X} \mathbf{R}\underline{\Gamma}_{[Y]}(M) \longrightarrow \mathbf{R}\underline{\Gamma}_Y(\mathcal{D}_X^\infty \otimes_{\mathcal{D}_X} M)$$

is an isomorphism.

In § 1, we recall some fundamental results concerning the duality for holonomic \mathcal{D}_X -Modules from Kashiwara and Kawai [2]. We also review, in § 2, the algebraic local cohomology of \mathcal{D}_X -Modules and an adjunction formula due to J.-P. Ramis. In the final section, we establish the theorem of equivalence described above, using the results recalled in the preceding sections.

In this note, "holonomic" will mean "coherent holonomic".

§ 1. Duality for holonomic \mathcal{D}_X -Modules. Let M be a bounded