22. *Remarks on the Deficiencies of Algebroid Functions of Finite Order*

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1. Introduction. Edrei and Fuchs [1] established the following interesting theorem:

Theorem A. *Let $f(z)$ be a meromorphic function of order λ, $0<\lambda<1$. Put*

$$u=1-\delta(0,f) \quad \text{and} \quad v=1-\delta(\infty,f), \quad 0\leqq u, v\leqq 1,$$

where $\delta(a,f)$ denotes the Nevanlinna deficiency of a value a. Then we have

$$u^2+v^2-2uv\cos\pi\lambda\geqq\sin^2(\pi\lambda).$$

Further, if $u<\cos\pi\lambda$, then $v=1$; if $v<\cos\pi\lambda$, then $u=1$.

This beautiful and elegant theorem solves completely the problem of finding relations between any two deficiencies of a meromorphic function of order less than one. A little later, Edrei [2] showed that the order λ in the theorem may be replaced by the lower order μ.

Shea [4] obtained a result which concerns with the Valiron deficiency $\Delta(a,f)$ instead of $\delta(a,f)$. That is, he proved

Theorem B. *Let $f(z)$ be a meromorphic function of order λ, $0<\lambda<1$, whose zeros lie on the negative real axis, and whose poles lie on the positive real axis. Put*

$$X=1-\Delta(0,f) \quad \text{and} \quad Y=1-\Delta(\infty,f).$$

Then, when $1/2\leqq\lambda<1$, we have

$$X^2+Y^2-2XY\cos\pi\lambda\leqq\sin^2(\pi\lambda).$$

When $0<\lambda<1/2$, the above inequality still holds provided

$$X\geqq\cos(\pi\lambda) \quad \text{and} \quad Y\geqq\cos(\pi\lambda).$$

The purpose of this paper is to extend these theorems to n-valued algebroid functions of order less than one. Our results are as follows:

Theorem 1. *Let $f(z)$ be an n-valued algebroid function of order λ, $0<\lambda<1$, defined by the irreducible equation*

(1.1) $$A_0(z)f^n+A_1(z)f^{n-1}+\cdots+A_n(z)=0,$$

where $A_0(z), A_1(z), \cdots, A_n(z)$ are entire functions without common zeros, and we suppose that 0 is not a Valiron deficient value for $A_0(z)$.

Let a_j, $j=1,\cdots,n$, be mutually distinct values, and put

(1.2) $$u_j=1-\delta(a_j,f) \quad \text{and} \quad v=1-\delta(\infty,f), \quad 0\leqq u_j, v\leqq 1.$$

Then, there is at least one a_ν, $1\leqq\nu\leqq n$, such that

(1.3) $$u_\nu^2+v^2-2u_\nu v\cos\pi\lambda\geqq n^{-2}\sin^2(\pi\lambda).$$